Bank Lending, Macroeconomic Conditions and Corporate Investment

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Keywords: investment, business cycles, collateral, loan rates, Dynamic choice
JEL Classification: C73, D83, G21

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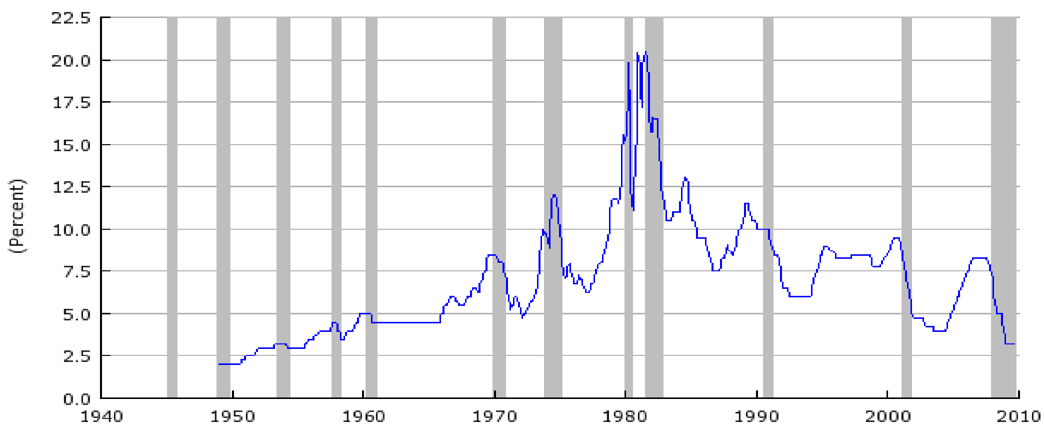
The main bank credit terms, the loan rate and collateral, are highly correlated with macroeconomic conditions. High (low) lending rates are typically accompanied with low (high) collateral requirements in economic upturns (downturns). This paper examines the impact of movements in loan rates and collateral on firm investment decisions within a dynamic model of bank lending and firm investment. We show that increased collateral induces firms to take on more (less) risk if the loan rate is higher (lower) than the expected investment return, and that an increase in the expected project arrival rate and a lowering in the expected investment return both increase firm risk taking. We also show that low quality firms are more sensitive to changing business conditions and loan contract terms. Our theoretical findings yield new predictions on the relationship between bank lending, corporate investment, and macroeconomic conditions.

1 Introduction

The main credit terms in bank lending, the loan rate and collateral, are highly correlated with macroeconomic conditions. As shown in Figure 1, the U.S. bank prime loan rate fluctuates roughly in a cyclical manner; for instance, it reached 8.2% in July 2007 before the financial crisis broke out but fell to 3.25% later in July 2009. Collateral, however, tends to be counter-cyclical, falling during economic expansions but rising during contractions (Asea and Blomberg 1998; Jimenez et al 2006). According to the Euro Area Bank Lending Survey (2009) conducted by the European Central Bank, around 25% of reporting banks expected to tighten their collateral requirements in the fourth quarter of 2007, and this number rose sharply a year later reaching 50% in the fourth quarter of 2008 when the financial crisis was spreading globally. The joint movements in the loan rate, collateral and macroeconomic conditions raise the following important questions: How does the combination of high (low) loan rate and low (high) collateral affect firm risk taking? How do changing investment opportunities interact with credit terms to affect firm investment and growth, and do such effects vary across firm? How do overall macroeconomic conditions affect the duration of credit relationship between banks and firms?

To address these questions, we present a new model of dynamic investment and financing decisions by firms under bank loan financing. In our model an entrepreneur decides whether or not to undertake a risky project given the loan rate and collateral requirements of banks and future investment opportunities. If the entrepreneur decides to take the project and is able to to repay the loan, the project continues into the next period. If the project fails, the collateral is seized by the bank and the entrepreneur has to wait for at least another period before investing in a new project. We derive the entrepreneur's optimal dynamic investment policy in terms of the type of risky project undertaken and show how the strategy is related to the loan rate, collateral and   expectations about furture macroeconomic conditions.

Figure 1: The U.S. Bank Prime Loan Rate: 1949-2009



Note: The shaded areas indicate U.S. recessions. The source of this figure is from the Board of Governors of the Federal Reserve System (2009 research.stlouisfed.org).

The formulation of the optimal investment policy of the bank-financed entrepreneur allows us to characterize the impacts of collateral and the loan rate on risk-taking incentives given uncertain future investment opportunities. We show that an increase in the loan rate always increases firm risk taking incentives while collateral has dichotomous effects on such incentives. In particular, in an economic boom if the loan rate rises above the expected future investment return, a rise in collateral induces firms to undertake riskier investments. However, in an economic contraction if the loan rate falls below the expected future investment return, a rise in collateral induces firms to select safer projects. Intuitively, when the loan rate is high, firms choose riskier projects so that the high cost of borrowing is offset by the advantage of limited liability; greater collateral exacerbates the high cost of debt and induces more risk taking. On the other hand, when the loan rate is low, firms take advantage of the low cost of borrowing by selecting safer projects and reducing the chance of deficit. More collateral increases the cost of default and makes the firm more prudent in its investment. Thus the effect of collateral on firm risk taking incentives depends on the level of the loan rate.

To the best our knowledge, this paper is the first to highlight theoretically the dichotomous effects of collateral.[1] Such effects have important implications for the structure of optimal loan contracts. Our analysis suggests that the *level* of the loan rate is critical in determining the impact of contract terms on risk taking. An increase in the loan rate transforms collateral from a risk curbing to a risk inducing instrument once the loan rate surpasses the expected investment return. Thus imposing a strict collateral requirement does not necessarily reduce firm risk taking since the effect of collateral depends on the level of the loan rate and expected investment return. Indeed, empirical observations showing that lower (higher) collateral policies are typically associated with high (low) loan rates in macroeconomic upturns (downturns) suggest that concerns about excessive borrower risk taking in high loan rate environments causes banks to avoid high collateral requirements.

We also show how changes in future investment opportunities affect firm investment decisions under bank financing. Future investment opportunities are characterized by the arrival rates of projects and expected investment returns. With higher arrival rates, an entrepreneur undertakes riskier projects as it becomes easier to find other projects if the original project fails. Also, with higher expected returns, the firm can maintain its profitability without recourse to riskier projects. In a macroeconomic upturn, project arrival rates and expected returns increase but as the upturn reaches its peak, expected returns decline while arrival rates remain high. Our theory predicts that such changed investment opportunities lead bank-financed firms to invest in riskier projects, resulting in a higher credit risk in the economy.

[1] A dichotomous effect of collateral is also found in a recent experimental study by Serra-Garcia (2010) who examines borrowers' effort choices in a one-period lending game. She shows that the effect of collateral on effort is strong when interest rates are low, but weak when these are high due to borrowers' loss aversion and fairness concerns (which offset the incentive effect of collateral).

We examine as well the impact of firm quality on the interaction between credit terms and risk taking. We define firm quality by the success probability distribution of iavailable investment projects where for a lower quality firm this distribution has a lower average but a higher variance. We show that the incentive effects of credit terms are stronger for lower-quality firms; thus, while a rise in the loan rate induces all firms to take on more risk, this effect is more pronounced for lower-quality firms. Moreover, at high loan rates, raising the collateral increases the investment risk of all firms but more so for lower-quality firms. On the other hand, at low loan rates, reducing collateral increases risk-taking incentives and more so for lower-quality firms. Overall, our results indicate that low-quality firms are more responsive to changing credit terms.

Finally, we analyze the duration of the bank-firm relationship, defined as the number of periods for a firm to successfully repay its debt. A good quality firm has a longer credit relationship with its banks, and a higher expected project return extends this duration. In contrast, higher project arrival rates induce firms to take riskier investments and shorten the relationship duration. To the extent that project arrival rates and expected returns vary with economic conditions, our theory suggests that the duration of the bank-firm relationship is sensitive to overall economic conditions.

Our theory yields several new empirical implications on the relationship between bank lending, macro-economic conditions and firm investment. It suggests that the relationship between the loan rate and collateral is nonlinear; that the loan rate and collateral are positively (negatively) correlated when the loan rates is below (above) the expected investment returns as collateral has dichotomous effects on firm risk taking. Moreover, the impact of macroeconomic conditions on risk taking depends on project arrival rates and their expected returns. Higher project arrival rates but lower expected returns lead to greater risk taking. Macroeconomic conditions also affect the duration of credit relationship between firms and banks; higher projects arrival rates and lower investment rates shorten the duration of a successful credit relationship. Finally, risk taking incentives of low quality firms are more sensitive to variations in macro-economic conditions.

In sum, the contributions of this paper are threefold. First, it introduces a new model of bank lending and firm investment. Extant models on the role of the loan rate and collateral in credit relationships are largely static.[2] This paper investigates the structure of loan contracts and their impact on firm investment decisions within a dynamic model of investment and lending. We show that the impact of collateral on firm investment depends on the level of the loan rate, the quality of firm, and expectations of future investment opportunities.

Second, our paper contributes to the literature on the impact of credit conditions on firm investments. The extant literature shows that the accessibility of credit has a significant impact on firm investment through collateral channel. For example, a monetary contraction for curbing inflation raises interest rates, lowers asset prices, and impairs collateral values, and thereby reduces investment and growth (e.g., Bernanke et al 1999). In contrast, asset price spikes under loose monetary policy increase the availability of assets pledged as collateral for loans, creating lending booms and stimulating firm investments (e.g., Goodhart et al 2005). Our results suggest an additional channel through which credit conditions affect firm investment, namely, that changing expectations of future economic conditions alter the impact of credit terms, the loan

[2] This literature consists of two broad explanations for the use of collateral as arising from *ex ante* private information and *ex post* incentive problems. The ex ante theories include: Stiglitz and Weiss (1981), Wette (1983), Chan and Kanatas (1985), Bester (1985) & (1987), Besanko and Thakor (1987 a & b), Chan and Thakor (1987), Boot et al (1991), etc. The ex post theories include Townsend (1979), Gale and Hellwig (1985), Williamson (1986), Innes (1990), Boot and Thakor (1994), Boyd and Simith (1994), Aghion and Bolton (1997), Holmstrom and Tirole (1997), Coco (1999), Albuquerque and Hopenhayn (2004), Cooley et al (2004), Besley and Ghatak (2009), etc. The role of collateral considered in these models is to mitigate agency problems on the part of the borrower. Inderst and Mueller (2007) consider the role of collateral in mitigating incentive problems on the part of the lender.

rate and collateral, on firm investment decisions.

Third, our model explains the relationship between collateral and macroeconomic conditions. Jimenez et al (2006) have found that macroeconomic conditions have a significant effect on the use of collateral; that collateral use decreases in economic upturns but increases in downturns. They note, however, "…little is known about the impact of macroeconomic conditions, such as the business cycle or monetary policy, on the decision to pledge collateral. … There is no theory to date on the relation between the use of collateral and macroeconomic conditions." Berger et al (2011a,b) point out that collateral is a widely used but not well understood debt contracting feature. Our paper fills this gap by offering a theory of the interaction between collateral and macro-economic conditions and generates predictions new empirical predictions.

The rest of the paper is structured as follows. Section 2 presents our model and derives the optimal investment decision rule for the firm. Section 3 conducts the comparative static analysis and examines the effect of credit terms on firm investment. Section 4 discusses the model's empirical implications. Section 5 concludes the paper.

2 The Model

2.1 Basic Setup

The entrepreneur can choose to be a worker earninging the market wage or to start a firm with limited liability. If she does the latter, she needs to choose a project among available alternatives and borrows from a bank to finance it, taking as given the credit contract terms pertaining to collateral provisions and interest payments. The acceptability of the project is affected by both its return/risk characteristics and the credit terms. If the project is not expected to be sufficiently profitable by the enterprenuer, she choose to be salaired as worker and waits for the arrival of a new project yielding a higher profit. If she accepts the project and finances it by borrowing from the bank, two possible outcomes emerge after one period: if the project succeeds, the firm and the bank continue their credit relationship; if the project fails, this relationship is terminated. In the latter case, the firm goes bankrupt and the entrepreneur becomes a salaried worker waits at least one period before starting a new firm. Waiting for one period provides a penalty for the failed entrepreneur to re-enter into entrepreneurship , e.g., due to the loss of reputation.[3]

As in the literature (e.g., Stiglitz and Weiss, 1981 and 1992; Ghatak 1999; Aronld and Riley, 2009), we assume that a project yields a random return $\mathbf{R}$ to the firm each period, producing $R(p)$ with probability p and zero with probability $p^c (\equiv 1 - p)$.[4] Projects have the same mean return per period: $E(\mathbf{R}) = pR(p) \equiv R_o$ for $\forall p \in \lfloor \underline{p},1 \rfloor$ and $\underline{p} > 0$. They differ in their riskiness, where a riskier project which succeeds produces a higher return for the firm, since $R'(p) < 0$.

A particular project type *p*, where *p* is its success probability, is drawn from the random variable $\mathbf{p}$ that is assumed to be $i.i.d.$ over time as in Burdett and Coles (1997). The distribution of project types $\mathbf{p} \in [\underline{p}(\mu),1]$ is $P(\mathbf{p} \le p) = G(p)$, with density $g(p)$, mean μ, and variance σ^2. This distribution is firm-specific, and depends on the firm's risk type $\mu^c (\equiv 1 - \mu)$. The population of projects available to the firm of quality type μ is $[\underline{p}(\mu),1]$ with $d\,\underline{p}(\mu)/d\mu^c < 0$. Thus the lower is its quality μ, the riskier is the firm.

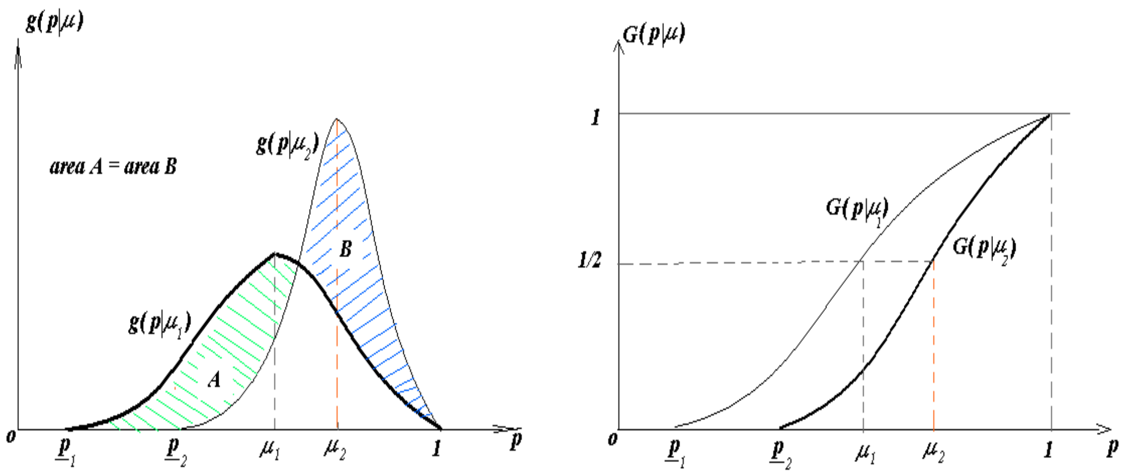
A riskier firm has a smaller mean probability of project success and a larger variance of project type, as shown in the left panel of Figure 2 under the unit-area preserving property of the

[3] Blanchflower, Oswald and Stutzer (2001) and Tam, Audretsch and Meijaard (2006) show that failed entrepreneurs tend to maintain their entrepreneurial preference and that it is a pervasive phenomenon for failed entrepreneurs to re-enter into entrepreneurship.

[4] For any variable *x* in this paper, we use the definition of $x^c \equiv 1 - x$ for notational simplicity.

density function (i.e., area A = area B). It follows from this that $\partial G(p \mid \mu)/\partial \mu < 0$ or $G(p \mid \mu_1) > G(p \mid \mu_2)$ for $\forall p < 1$ and $\mu_1 < \mu_2$, as depicted in the right panel of Figure 2. We shall at times suppress μ from $\underline{p}$, g, and G to simplify notation.

Figure 2: The Distribution of Project Types



The firm carries out one project per period by borrowing from a bank one unit of credit. Old debt borrowed for a successful project has to be repaid before new debt is serviced in order to continue the project. If the firm defaults on its loan because of project failure, it loses its pledged collateral to the bank. The loan rate and collateral are variables observable to the firm and the bank.

The random profit $\boldsymbol{\pi}$ to the firm each period is given by $R(p) - r$ with probability p, and $0 - c$ with probability p^c, where r is the gross interest rate (= 1 + interest rate) paid by the firm if the project succeeds and c is the value of collateral claimed by the bank if the firm defaults. The mean profit per period is:

$$E(\boldsymbol{\pi}) = R_o - \left(pr + p^c c\right) \equiv \pi_e(p). \tag{1}$$

Note that $r > 1$ but $c \leq$ or > 1 for one unit of loan. Our model covers cases of over-collateralization, full-collaterization, and under-colleralization; $c > r$, $c = r$, and $c < r$. The average recovery rate of defaulted bank loans in the U.S. is relatively high. For example, Moody's reports that the average recovery rate for bank loans in the U.S. was 71% between 1990 and 2006, a period that included long economic expansions and two recessions. The loan recovery rate tends to deteriorate in economic downturns. Fitch Ratings shows that the recovery rate on U.S. bank loans was only 57.5% in first six months of 2009. Our anlysis focuses on the realistic case of under-collateralization, i.e., $c < r$, as in Stiglitz and Weis (1983) and Arnold and Riley (2009). The above discussion leads to the following proposition:

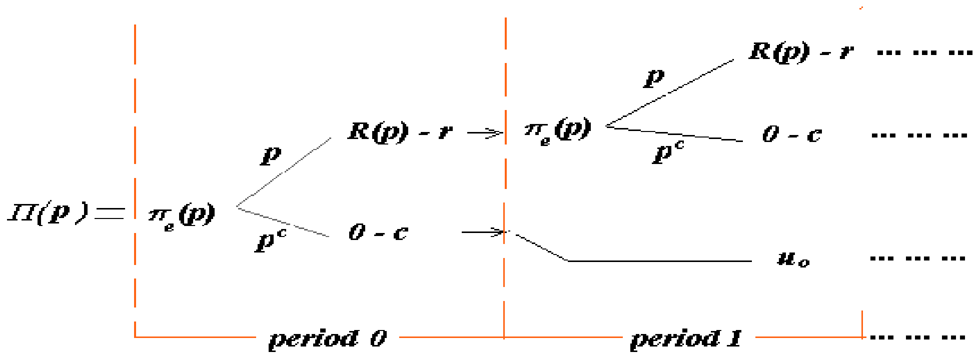
Proposition 1:

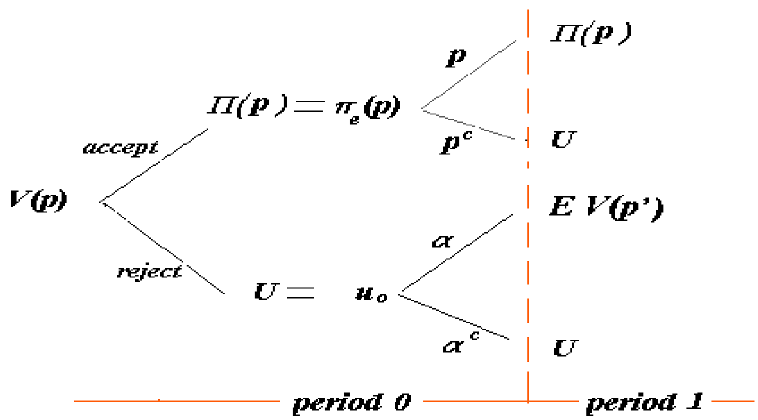
Riskier projects are more profitable to the firm given a credit contract (c, r) *with under-collateralization, since* $c < r$ *implies* $\pi_e'(p) = -(r - c) < 0$*; that is a project of lower type p yields a higher mean profit* $\pi_e(p)$ *per period.*

2.2 Dynamic Project Selection

The investment choice of the entrepreneur is formulated in the dynamic model described next. Consider firm value from project selection over discrete intervals of time. $V(p)$ denotes the present value of the decision as to whether or not to undertake a project of type p this period, given possible investment opportunities in subsequent periods. The decision is made at the beginning of period 0 and takes into account of future risks and opportunities. The project selection value is specified as $V(p) = \max\{\Pi(p), U\}$ to assess trading and investment opportunities over time, where $\Pi(p)$ and U are defined shortly. $V(p)$, together with its componets, is illustrated in Figure 3.

Figure 3: The Dynamics of Firm Investments





$\Pi(p)$ denotes the present value of the firm undertaking a type p project this period and behaving optimally in the fure. This value is determined by a recursive equation:

$$\begin{aligned} \Pi(p) &= p\left\{\left[R(p)-r\right]+\beta\Pi(p)\right\}+p^{c}\left\{-c+\beta U\right\}, \\ \Pi(p) &= \pi_e(p)+\beta\left[p\Pi(p)+p^{c}U\right], \end{aligned} \qquad (2)$$

where Equation (1) has been used and $0<\beta<1$ is a discount factor reflecting the entrepreneur's time preference. If the project is a success and produces a profit of $\left[R(p)-r\right]$ (with probability p) this period, it will continue to the next period and the process generating $\Pi(p)$ will be repeated. If the project fails and collateral c is seized by the bank (with probability p^c) this period, the entrepreneur will wait at least one period for another project and will receive a waiting value U from next period onwards.

Rearranging Equation (2) yields:

$$\Pi(p) = \frac{\pi_e(p) + \beta p^c U}{1 - \beta p}. \tag{3}$$

As derived in Appendix (I), and depicted in Figure 4, $\Pi(p)$ decreases in *p* at an increasing rate since $\Pi'(p) < 0$ and $\Pi''(p) < 0$. This suggests that the firm will accept a riskier project given the credit terms.

U denotes the value of waiting for the entrepreneur who encounters no project this period or rejects the currently available project and waits for a better future opportunity. She receives the reservation payoff u_o in each waiting period, and projects are assumed to arrive with probability α. If a type $\mathbf{p}$ project arrives (with probability α) in the next period, the entrepreneur decides whether to undertake it and receive the new value $V(\mathbf{p})$. If no project arrives (with probability $\alpha^c \equiv 1 - \alpha$), she continues to wait and receives U from then on.

Thus U is given by a recursive equation:

$$U = u_o + \beta\left[\alpha EV(\mathbf{p}) + \alpha^c U\right], \tag{4}$$

$$\text{or } U = \frac{u_o + \alpha\beta EV(\mathbf{p})}{1 - \alpha^c \beta},$$

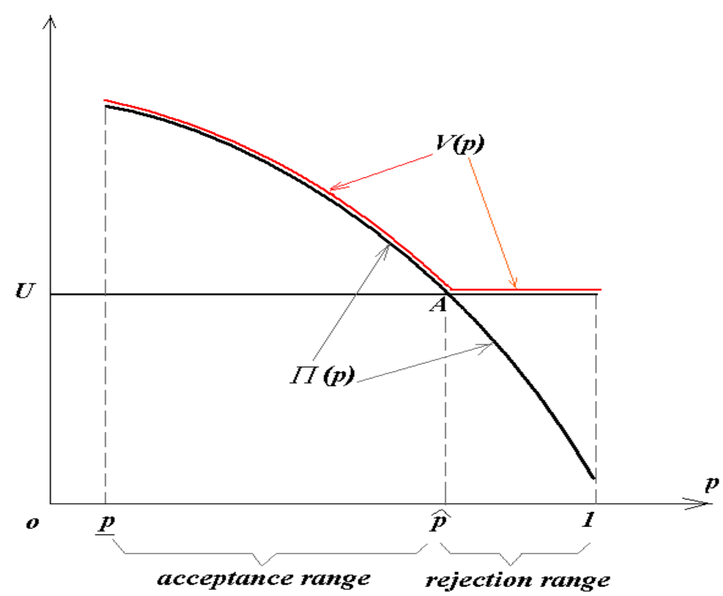
where the expectation *E* is taken with respect to *G* which captures the project uncertainty faced by the firm. *U* is constant in *p* as shown in Figure 4 after averaging out the randomness of $\mathbf{p}$.

Substituting Equations (4) and (3) into the firm's value function $V(p)$ yields Bellman's functional equation:

$$V(p) = \max\left\{\frac{\pi_e(p) + \beta\left[p^c u_o - \alpha^c \pi_e(p)\right] + \beta^2\left[p^c \alpha EV(\mathbf{p})\right]}{(1 - \beta p)(1 - \alpha^c \beta)}, \quad \frac{u_o + \alpha\beta EV(\mathbf{p})}{1 - \alpha^c \beta}\right\}, \tag{5}$$

which is used to derive the firm's reservation project type $\hat{p}$ (≤ 1). The investment policy rule is: accept any project of type *p* without delay if $p \leq \hat{p}$; reject any project of type *p* if $p > \hat{p}$ and wait for better opportunities. This optimal project selection strategy is depicted in Figure 4, with $\Pi(p) > U$ (or $NPV \equiv \Pi - U > 0$) for $p < \hat{p}$, and $\Pi(p) < U$ for $p > \hat{p}$. We label this reservation policy the dynamic NPV rule for investment choice: the firm accepts a project if its present value, *Π*, is greater than the reservation value *U* of waiting.

Figure 4: Optimal Choice between Investing Now orWaiting for Future Opportunities



Equation (5) models the firm's investment decision in a dynamic framework, where future opportunities are taken into account in making a decisions at the begining of period 0. The optimality condition for this model is derived in Appendix (II) and is:

$$\frac{R_o - u_o - c}{r - c} = \hat{p} + \alpha\beta(1 - \beta\hat{p}) \int_{\underline{p}}^{\hat{p}} \frac{G(p \mid \mu)\,dp}{(1 - \beta p)^2} > 0, \tag{6}$$

which defines the reservation project type $\hat{p}$ (≤ 1) as a function of ($c, r; \mu$) among other things (i.e., $R_o - u_o$, α, β).

A type p project is acceptable to firm μ at the loan contract (c, r) only if $p \in \left[\underline{p}, \hat{p}\right]$ where $\hat{p} = \hat{p}(c, r; \mu)$, so that $\hat{p}$ matters for firm riskiness. If $\hat{p}$ increases, the project acceptance range $\left[\underline{p}, \hat{p}\right]$ becomes wider and firm investment becomes more prudent since projects that were rejected before now become acceptable. Conversely, the firm's project choice gets riskier if $\hat{p}$ falls. Without loss of generosity, henceforth we normalize u_o *as* 0 for simplicity.

2.3 Comments on the Optimality Condition

Equation (6) can be rewritten as:

$$\frac{R_o - c}{r - c} = \hat{p}_m, \tag{7}$$

where $\hat{p}_m \equiv \hat{p} + \hat{p}_f$ and $\hat{p}_f \equiv \alpha\beta(1-\beta\hat{p})\int_{\underline{p}}^{\hat{p}} \frac{G(p)\,dp}{(1-\beta p)^2} > 0$. Note that $\hat{p}$ (≤ 1, by definition) is the threshold probability in period 0 under the dynamic NPV rule; it denotes the success probability of the marginal project. In addition, $\hat{p}_f$ [≤ 1, see Equation (A7)] is the total future probability of project success across periods $1,2,3,\cdots$ (allowing for the risk of project failure in future periods).

The probability $\hat{p}_f$, simplified in Appendix (III), is:

$$\hat{p}_f = \alpha\hat{G}\left[1-(1-\beta\hat{p})\mu_{T_d}\right] \equiv \alpha(1-\hat{q}_1)\hat{G}, \tag{8}$$

where $\hat{G} = G(\hat{p}) = \Pr(p \leq \hat{p})$ with μ omitted. $\mu_{T_d} = E_{p\leq\hat{p}}(1-\beta p)^{-1}$ is the "discounted" mean duration of successfully operating an accepted project over periods $1,2,3,\cdots$.[5] $\hat{q}_1$ [$\equiv (1-\beta\hat{p})\mu_{T_d}$]

[5] (βp) is regarded as the "discounted" probability of success in running project p since discounting the return value

is the "discounted" failure rate of the project after its successful operation for μ_{T_d} periods. Then $\left(1-\hat{q}_1\right)$ is the duration-adjusted probability of the project operating successfully in future periods $1,2,3,\cdots$. Note that $\alpha\hat{G}$ is the joint probability that the project is available and acceptable. Hence, $\hat{p}_f = \alpha\hat{G}\left(1-\hat{q}_1\right)$ is the total future probability that a project is available**,** acceptable**,** and successful (until failure occurs after survival for μ_{T_d} periods).

A variant of Equation (8) derived in Appendix (III) is helpful for interpreting $\hat{p}_f$.

$$\hat{p}_f = \beta\alpha\hat{G}E_{p\le\hat{p}}\left[\left(\frac{1}{1-\beta p}\right)(\hat{p}-p)\right], \tag{9}$$

which indicates that $\hat{p}_f$ is the product of discount rate β, project arrival rate α, project acceptability rate $\hat{G}$, project survival rate $\left(1-\beta p\right)^{-1}$, and project preferability rate $\left(\hat{p}-p\right)$. For a given $\hat{p}$, a lower type p project or a larger $\left(\hat{p}-p\right)$ is preferred by the firm. Thus $\hat{p}_f$ is the "discounted" probability that a project is available, acceptable, preferable, and sustainable.

Given $\hat{p}$ and $\hat{p}_f$ as the current and future probabilities of project success, respectively, $\hat{p}_m = \hat{p}+\hat{p}_f$ is the sum of project success probabilities for **all periods,** and can be less than, equal to, or greater than one. That is, the possibility that an accepted project continues into the infinite future allows the sum of its per-period success probabilities to be equal to or greater than

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$R_o[= pR(p)]$ is equivalent in effect to discounting the success probability *p*; i.e., $\beta R_o = (\beta p)R(p)$. Intuitively, a successfully run future project is less valuable in terms of *present value* than a successfully run current project of the same type *p* (with the nominally equal returns $R(p)$ in both periods). Also, βp is the parameter of the random variable $T_d' = \{1,2,3,...\}$ which indicates that the project will fail in period T_d' after its successful operation in the first ($T_d'-1$) periods. This random variable follows a geometric distribution: $\Pr(T_d' = t) = (\beta p)^{t-1}(1-\beta p)$ and has the expected value: $E(T_d') = \sum_{t=1}^{\infty} t\left(\beta p\right)^{t-1}(1-\beta p) = 1/(1-\beta p)$. Note that $E(T_d')$ is conditional on the acceptability of project type *p*. Then, $\mu_{T_d} = E_{p\le\hat{p}}[E(T_d')]$ is the average duration of an accepted project's successful operation.

unity.[6] This sum will become even greater with a bigger chance of *reincarnation* wherein a new acceptable project arrives to replace the old project after it has failed.

Equation (7) can be changed to the standard marginal condition (*MR* = *MC*) as follows:

$$R_o = r\hat{p}_m + c\hat{p}_m^c, \tag{10}$$

where $\hat{p}_m^c \equiv 1 - \hat{p}_m$. In Equation (10), the marginal return on investment, $MR = R_o$, is the expected return from a project, $R_o = E(\mathbf{R})$. The marginal cost of investment, $MC = r\hat{p}_m + c\hat{p}_m^c$, is the expected cost of borrowing and consist debt repayments in case of success and collateral losses in case of failure. Therefore, the threhold success probablitiy $\hat{p}$ of the marginal project used in period 0 to decide whether to accept a type *p* project is determined by the marginal condition in Equation (10), where $\hat{p}_m$ is a function of $\hat{p}$ as in the definition in Equation (7).

3 Comparative Static Analysis

We now conduct comparative static analyses to study optimal changes in the firm's dynamic project choices with respect to credit contract terms, future investment opportunities, and firm risk type.

3.1 Credit Terms

We first examine the impacts of the loan rate *r* and of collateral *c* on the firm's optimal rule $\hat{p}$ for project selection. Differentiating the reservation project type $\hat{p}$ in Equation (6) with respect to *r* and *c* yields:

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[6] To see this, it suffices to look at a simplified numerical example where an accepted project can be continued for multiple periods but with an equal and *independent* chance p (< 1) to fail in each period. Using a decision-tree technique yields the sum p_m of probabilities that the project happens to be run successfully in all periods: $p_m = p + p^2 + p^3 + ... = p / p^c$. Thus $p_m = 3/7 < 1$ for a low $p = 0.3$ but $p_m = 1.5 > 1$ for a high $p = 0.6$. On average, however, the survival duration of a type *p* project is only $1/p^c$ periods (not forever).

$$\frac{\partial \hat{p}}{\partial r} = -\frac{R_o - c}{(r-c)Y} < 0$$

$$\frac{\partial \hat{p}}{\partial c} = \frac{R_o - r}{(r-c)Y} \begin{cases} \leq 0, & if\ r \geq R_o \\ > 0, & if\ r < R_o \end{cases} \tag{13}$$

$$\frac{\partial^2 \hat{p}}{\partial r \partial c} = \frac{1}{(r-c)Y}\left[\frac{\alpha\beta\hat{g}}{1-\beta\hat{p}}\frac{(R_o - r)(R_o - c)}{Y^2} - \frac{2R_o - (r+c)}{r-c}\right] < 0$$

where $\hat{g} \equiv g(\hat{p})$, $Y \equiv (r-c)(1+\alpha\beta\mu_{T_d}\hat{G}) > 0$, $2R_o > r + c$, and $r > c$. These results imply the following proposition:

Proposition 2:

A. *A higher loan rate induces the firm to make a riskier investment decision by decreasing its reservation project types (i.e.,* $\partial\hat{p} / \partial r < 0$ *).*

B. *More collateral causes the firm to select riskier project (i.e.,* $\partial\hat{p} / \partial c < 0$ *) if the loan rate is higher than the project return (* $r > R_o$ *), but makes it more prudent (i.e.,* $\partial\hat{p} / \partial c > 0$ *) if the loan rate is lower than that return (* $r < R_o$ *).*

C. *The incentive effect of the loan rate becomes stronger with more collateral (i.e.,* $\frac{\partial}{\partial c}\left|\frac{\partial\hat{p}}{\partial r}\right| > 0$ *), as depicted in Figure 5.*[7]

Proof: See Appendix (IV).

Proposition 2 shows that while a higher loan rate always creates a moral hazard problem (part *A*), the impact of collateral on borrower risk-taking incentives is dichotomous in that increasing collateral has a positive incentive effect if the loan rate is low, and a negative effect if

[7] The operator of absolute value used here and below indicates only the magnitude of derivatives, which is not inconsistent with the fact that $|x|$ is not differentiable at $x = 0$. However, $|x|$ is differentiable at $\forall x \neq 0$.

the loan rate is too high (part *B*). The reasons for this dichotomy are discussed next.

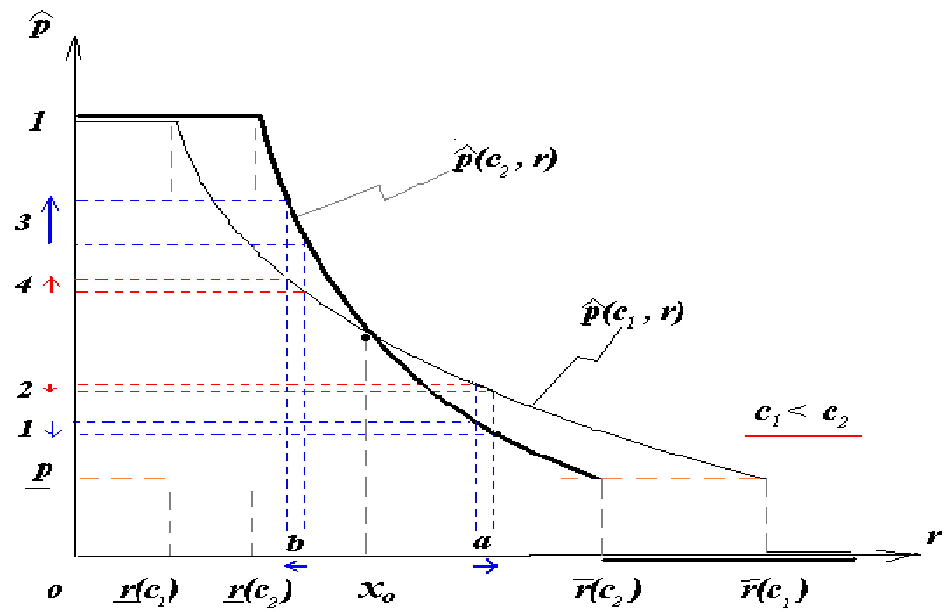
At low loan rates ($< R_o$), firms try to make best use of low borrowing costs and high potential profits by selecting safe projects and avoiding investment failures that might engender collateral losses. Given that there is a sufficient surplus for the firm if it is successful, it has no incentive to increase its risk taking. More collateral increases the cost of default and induces the firm to make more prudent investments. At high loan rates ($> R_o$), it is difficult to profit from a safe project with a low return and the firm will choose a sufficiently risky project to offset its high debt cost. Since the likelihood of default is high in this case, the firm takes advantage of limited liability. Increasing collateral provisions exacerbates the already high cost of debt and induces more risk taking.

The interaction between the two credit terms is such that greater collateral generates investment decisions that are more sensitive to interest rate changes. While a fall (rise) in the loan rate induces firms to undertake safer (riskier) projects, this effect is strengthened by a rise in collateral (part *C* in Proposition 2). On the one hand, the negative effect of more collateral aggravates the negative effect of a higher loan rate when the latter is already high ($r > R_o$); this is depicted in Figure 5 where for a rise in r (shown by arrow a) and a rise in c (in the form of $c_2 > c_1$)), arrow 1 is worse under c_2 than is arrow 2 under c_1, as measured by drops in $\hat{p}$ (i.e., higher risk). On the other hand, the positive effect of more collateral reinforces the positive effect of a lower loan rate when the latter is already low ($r < R_o$); this is illustrated in Figure 5 where for a decline in r (shown by arrow *b*) arrow 3 is better with c_2 than is arrow 4 with c_1, as measured by increases in $\hat{p}$ (i.e., lower risk).

The above results indicate that collateral affects firm investment incentives both directly and indirectly. The direct or **first-order** effect is measured by the second expression in Equation

(13) and is shown by the vertical gap between the two curves (with separate **locations**) in Figure 5. When the loan rate is greter than R_o, imposing lower collateral has a positive incentive and reduces the moral hazard problem. When the loan rate is lower than R_o, tightening collateral requirements induces firms to make safer investments and reinforces the positive incentive effect of the lower loan rate.

Figure 5: The Effects of the Loan Rate and Collateral on Investment Choice



The indirect or **second-order** effect of collateral arises through its interactions with the loan rate. This is captured by the third expression in Equation (13) and is reflected in the differing **slopes** of the two curves in Figure 5. A higher loan rate (> R_o) becomes less harmful if collateral is lower, as implied by arrow 2 which is shorter than arrow 1 in Figure 5. A lower loan rate (< R_o) becomes more beneficial for loan repayment if collateral is greater, as suggested by arrow 3 which is larger than arrow 4 in Figure 5. This implies that it is desirable for banks to reduce collateral during booms and to raise it during slumps, as the loan rate changes over the business cycle. Our theoretical predictions pertainting to direct and indirect effects offers a potential explanation for the observed cyclical patterns of credit terms in bank lending across the business cycle: high loan rates are associated with low collateral in upturns and the reverse occurs in downturns.

3.2 Firm Quality

The effect of firm type on project selection and its interaction with credit terms are presented in the following:

$$\frac{\partial \hat{p}}{\partial \mu} = -\frac{r-c}{Y}\left\{\alpha\beta(1-\beta\hat{p})\int_{\underline{p}}^{\hat{p}} \frac{\partial G(p|\mu)}{\partial \mu}\frac{dp}{(1-\beta p)^2}\right\} > 0$$

$$\frac{\partial^2 \hat{p}}{\partial \mu \partial r} = \frac{R_o - c}{(r-c)Y^2}\frac{\partial Y}{\partial \mu} > 0 \tag{14}$$

$$\frac{\partial^2 \hat{p}}{\partial \mu \partial c} = -\frac{R_o - r}{(r-c)Y^2}\frac{\partial Y}{\partial \mu}\begin{cases} \geq 0, & \text{if } r \geq R_o \\ < 0, & \text{if } r < R_o \end{cases},$$

where $\frac{\partial Y}{\partial \mu} > 0$ since $\frac{\partial G}{\partial \mu} < 0$. These results are summarized as follows.

Proposition 3:

A. *Lower-quality firms (i.e., higher μ^c) undertake riskier projects (i.e., $\partial \hat{p} / \partial \mu^c < 0$).*

B. *Decreasing (increasing) loan rates lead firms to choose safer (riskier) projects (i.e., $\partial \hat{p} / \partial r < 0$), and this effect is stronger for lower-quality firms (i.e., $\frac{\partial}{\partial \mu^c}\left|\frac{\partial \hat{p}}{\partial r}\right| > 0$).*

C. *When loan rates are high ($r > R_o$), increasing collateral force lower-quality firms to take higher risk (i.e., $\frac{\partial}{\partial \mu^c}\left|\frac{\partial \hat{p}}{\partial c}\right| > 0$ with $\frac{\partial \hat{p}}{\partial c} < 0$). When loan rates are low ($r < R_o$),* *decreasing collateral causes lower-quality firms to become riskier in their investment (i.e.,* $\frac{\partial}{\partial \mu^c}\frac{\partial \hat{p}}{\partial c} > 0$ *with* $\frac{\partial \hat{p}}{\partial c} > 0$*).*[8]

Proof: See Appendix (V).

Proposition 3 asserts that lower-quality firms have stronger incentives for risk taking (part *A*), and that their investment decisions are more responsive to changes in the loan rate and collateral (parts *B* and *C*). Specifically, a rise in the loan rate induces all firms to take on more risk, with lower-quality firms taking on far more risk. Firm quality also affects the incentive effect of collateral which also depends on the interest rate. When the rate is high (> R_o), stricter collateral requirements induce greater risk taking by low-quality firms than their high-quality counterparts. When loan rate is low (< R_o), the imposition of lower collateral increases risk-taking incentives of low quality firms more than of their high-quality counterparts. We found before that collateral has dichotomous effects on risk taking, and we now show that such effects are stronger for lower-quality firms.

3.3 Investment Opportunities

Future investment opportunities are indicated by the project arrival rate α and expected project return R_o. These indicators vary over the business cycle, and their impact on project selection is as follows:

$$\frac{\partial \hat{p}}{\partial \alpha} = -\frac{r-c}{Y}\left(1-\hat{q}_1\right)\hat{G} < 0 \qquad (15)$$

$$\frac{\partial \hat{p}}{\partial R_o} = \frac{1}{Y} > 0$$

[8] The interaction of firm risk type μ^c with bank credit terms $(c, r) \equiv L$ can be compactly written in a unified form: $\frac{\partial}{\partial \mu^c}\left|\frac{\partial \hat{p}}{\partial L}\right| > 0$.

$$d\hat{p} > \frac{r-c}{Y}\left(\frac{c}{r-c}+\hat{p}\right)\frac{d\alpha}{\alpha} > 0 \text{ if } \frac{dR_o}{R_o} > \frac{d\alpha}{\alpha} > 0$$

$$d\hat{p} < \frac{r-c}{Y}\left(\frac{c}{r-c}+\hat{p}\right)\frac{d\alpha}{\alpha} < 0 \text{ if } \frac{dR_o}{R_o} < \frac{d\alpha}{\alpha} < 0 \text{ or } \frac{|dR_o|}{R_o} > \frac{|d\alpha|}{\alpha}.$$

where $d\hat{p}$ is the total differential of $\hat{p}$ caused by changes in both R_o and α.[9] The results in Equation (15) imply the following proposition.

Proposition 4:

A. *Firms select riskier (safer) projects when future project availability is greater (lower), i.e., a larger α causes a lower $\hat{p}$.*

B. *Firms select safer (riskier) projects when the expected return on investment is higher (lower), i.e., a larger R_o leads to a higher $\hat{p}$.*

C. *Firms become more prudent in their investment (i.e., $d\hat{p} > 0$) if the increass in expected return dominate the increase in the project arrival rate(i.e., $dR_o/R_o > d\alpha/\alpha > 0$), whereas firms become riskier (i.e., $d\hat{p} < 0$) if the decline in expected return is larger than the decline of the of project arrival rate(i.e., $|dR_o|/R_o > |d\alpha|/\alpha$).*

Proof: See Appendix (VI).

Proposition 4 illustrates the impact of future investment opportunities on firm investment choices. Greater project availability increases investment options and makes firms more selective about project type (part *A*). With a higher project arrival rate, a firm chooses a riskier project since it is easier to find another one if a project fails. Higher expected returns on investment generate larger profit margins and reduce risk-taking (part *B*). A firm that is more optimistic

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[9] *dz/z* denotes a (positive or negative) percentage change in any variable *z*.

about investment returns selects safer projects and, all else equal, stays in business longer because it avoids risky projects that are more likely to fail.

The above results show that the two investment-opportunity indicators, α and R_o, have opposing effects on investment. Their net impact depends on which factor is dominant in a particular situation (part *C*). At the early stage of an economic boom, with rising project arrival rates and investment returns, the net effect on firm risk taking is ambiguous. As the economy approaches the peak of the boom, project arrival rate may be sustained while investment returns decline as diminishing marginal returns sets in. Firms then have strong incentives to invest in riskier projects and credit risk accumulates before the termination of the boom. Therefore, a higher interest rate and a lower investment return both serve to reinforce moral hazard problems, resulting in loan defaults as the economy plunges into a recession.

3.4 Relationship Duration

Firm operates a successful project over multiple periods and we now examine the duration of a successful project. If the firm waits *T* periods before encountering (with probability α) an acceptable project (with probability $\hat{G}$), the random variable *T* follows a geometric distribution with parameter $\alpha\hat{G}$ and mean $\mu_T = E(T) = 1/(\alpha\hat{G})$. Similarly, the random duration T' of successful operation of an accepted project has a mean of $\mu_{T'} = E(T') = 1/\tilde{p}^c$; as the project may fail with an average risk $\tilde{p}^c (\equiv 1-\tilde{p})$ after T' periods of operation, where $\tilde{p} = E_{\underline{p} \le p \le \hat{p}}\left(p\right)$ $= \tilde{p}\left(c, r \middle| \mu\right)$. $\tilde{p}$ is the average success probability of a project acceptable to the firm type μ, or its average repayment rate under the loan contract (*c, r*). One can show that $\tilde{p}$ is related positively to $\hat{p}$ with $\underline{p} < \tilde{p} < \min\left(\hat{p}, \mu\right)$. Note that $\mu_{T'}$ is the mean duration of the credit relationship before default. It is easy to see that:

$$\frac{\partial \mu_T}{\partial(\mu, R_o)} < 0, \qquad \frac{\partial \mu_{T'}}{\partial(\mu, R_o)} > 0, \qquad \frac{\partial \mu_{T'}}{\partial \alpha} < 0 \tag{16}$$

These results generate the following proposition.

Proposition 5:

A. *A higher-quality firm (with larger μ) has a shorter waiting duration (smaller μ_T) for an acceptable project, but it has a longer duration (larger $\mu_{T'}$) of a successful credit relationship with its financing bank.*

B. *Higher expected project returns (higher R_o) shorten waiting duration (smaller μ_T), but extend the duration of the credit relationship (larger $\mu_{T'}$) because of lower risks assumed by investing firms.*

C. *Higher project arrival rates (larger α) make firms select projects with greater risk and shorten the duration (smaller $\mu_{T'}$) of the credit relationship.*

Proof: See Appendix (VII).

Proposition 5 pertains to the impact of firm type, and of investment opportunities, on the length of waiting time for an acceptable project and the maintainance of a successful credit relationship. A lower-quality firm will choose a risker project and have to wait longer for an acceptable investment opportunity, but may maintain a shorter-term credit relationship with its financier since its project is riskier with greater chance of failure (part *A*). A lower prospect of earnings growth prolongs the waiting time for an acceptable project, but shortens the duration of the credit relationship because of a greater chance of project failure (part *B*). A higher project arrival rate makes the firm more selective in its investments, and its relationships with the bank is of shorter duration because of a higher likelihood of default (part *C*). These results highlight the sensitivity of the duration of the bank-firm relationship to economic conditions. In an economic downturn, for example, earnings growth declines and R_o become smaller. Firms turn to riskier investments to maintain their profitability and their relationships with banks become less stable and of shorter duration. This is more pronounced for lower-quality firms.[10]

4 Empirical Implications

Our theoretical analysis of the firm's dynamic project choices given credit contract terms, future investment opportunities, and firm risk type yields several new empirical implications. First, the relationship between loan rate and collateral is nonlinear in that loan rates are positively (negatively) related to collateral when the loan rate is below (above) the expected investment return. This is because increasing (decreasing) collateral mitigates risk taking incentives engendered by increasing loan rates when the loan rate is below (above) the expected investment return. Our results are consistent with observed time series pattern showing that low collateral is associated with high loan rates in economic expansions, whereas high collateral is associated with low loan rates in recessions. However, the cross-sectional firm-level evidence on the relationship between the loan rate and collateral is mixed. For example, Berger and Udell (1990) find a positive relation between the use of collateral and debt yields, while Knox (2005) shows that collateral is negatively related to firm borrowing costs. Our results suggest that collateral has dichotomous effects depending on the level of the loan rate. This nonlinear relationship between the loan rate and collateral provides a potential reconciliation of the seemingly conflicting empirical evidence.

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[10] Since $\frac{\partial^2 \hat{p}}{\partial \mu^c \partial R_o} = \frac{1}{Y^2}\frac{\partial Y}{\partial \mu} > 0$, the reservation project types $\hat{p}$ of lower-quality firms are more sensitive to changes in the expected project return R_o. In bad times with R_o falling and all firms getting riskier in their investments, lower-quality firms become even riskier and hence have much shorter credit relationships as confirmed by $\frac{\partial \mu_{T'}}{\partial \mu^c} < 0$.

Second, our theory suggests that macroeconomic conditions, indicated by project arrival rates and expected investment returns, affect risk taking incentives of bank-financed firms. A related study by Coles et al (2006) finds that firms with greater market-to-book ratios have lower default risk. To the extent that the market-to-book ratio reflects both project arrival rates and expected investment returns, their findings suggest that macroeconomic conditions and firm risk are negatively correlated. Our theory predicts that project arrival rates and expected investment returns have opposite effects on firm risk taking, it would be interesting to disentangle their effects on firm risk taking to provide further insights on the impact of macro-economic conditions in bank-financing firms.

Our theory also predicts that higher project arrival rates and lower expected investment returns increase risk-taking and shorten the duration of a successful credit relationship between the firm and the bank and, also, that lower-quality firms choose riskier investments and have shorter-term credit relationships with their lenders. Several studies have explored the determinants of relationship-duration in bank lending. For example, Ongena and Smith (2001a) find that longer duration implies a higher the probability of termination so that to the extent that longer duration is associated with a greater likelihood of an economic downturn, their finding is consistent with our theory. Farinha and Santos (2002) find that small and young firms maintain the shortest relationships; since such firms typically have lower credit ratings and higher default probabilities, their finding suggests, consistent with our theory, that lower quality firms have shorter relationship duration with lenders.

5 Conclusion

We presented a dynamic model of the interaction between bank lending, firm investment, and macroeconomic conditions. We showed that, although higher loan rates increase firm incentives for risk taking, collateral has dichotomous effects on investment behavior depending on the loan rate: increased collateral induces firms to take on more (less) risk if the loan rate is higher (lower) than the expected investment return. Moreover, the incentive effect of the loan rate becomes stronger with more collateral. Firm quality also has an important bearing on risk taking; as the interest rate rises in an economic upturn, firms choose riskier projects (especially lower-quality firms). When the interest rate rises above the expected investment return, a higher level of collateral exacerbates moral hazard problems (especially for lower-quality firms). If the interest rate is lower than the expected investment return, higher collateral mitigates moral hazard problems especially for lower-quality firms. Given credit contract terms, a higher expectation of project arrivals or lower project expected returns increase firm risk-taking incentives.

Our model generates theoretical results that are consistent with extant empirical findings and yield new predictions. To make the model tractable, we have made some simplifying assumptions. Several extensions for future research emerge. For example, our model did not allow for firm self-financing and it would be interesting to investigate the firm's profit retention decision. While we considered the interaction between credit terms and their effects on firm investment, there was no feedback effect from firm investment to bank credit terms. A natural extension would be to analyze the robustness of our results by endogenizing credit terms in a general equilibrium framework. These modifications, in conjunction with the integration of dynamic firm investment with bank lending, would provide a fruitful avenue for future research.

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Appendix

(I) Prove the signs of $\Pi(p)$'s derivatives in Equation (3)

Point *A* in *Figure 4* shows $\Pi(\hat{p}) = U$ as a cutoff condition. Substituting from Equation (3) to this condition yields:

$$\beta^c U = \pi_e(\hat{p}) \ (= R_o - \hat{p}r - \hat{p}^c c), \tag{A1}$$

where $\beta^c \equiv 1-\beta$ and $\hat{p}^c \equiv 1-\hat{p}$. Change Equation (A1) to $R_o - c - \beta^c U = \hat{p}(r-c)$. Multiplying $(-\beta)$ first, then adding $(r-c)$, and finally arranging terms, yields:

$$r - c - \beta(R_o - c - \beta^c U) = (1-\beta\hat{p})(r-c). \tag{A2}$$

Differentiating Equation (3) with respect to p yields:

$$\Pi'(p) = \frac{\beta(R_o - c - \beta^c U) + c - r}{(1-\beta p)^2}.$$

Combining this with Equation (A2) yields:

$$\Pi'(p) = -\frac{(r-c)(1-\beta\hat{p})}{(1-\beta p)^2} < 0, \tag{A3}$$

$$\Pi''(p) = -\frac{2\beta(r-c)(1-\beta\hat{p})}{(1-\beta p)^3} < 0,$$

since $c < r$, $\beta < 1$, and $\hat{p} < 1$.

(II) Derive the optimality condition in Equation (6)

Change the second expression in Equation (4) to:

$$\left[(1-\alpha^c\beta)U - u_o\right]/(\alpha\beta) = EV(p). \tag{A4}$$

Calculate the expected value of $V(p) = \max\{\Pi(p), U\}$ in the following manner:

$$EV(p) = \int_{\underline{p}}^{\hat{p}} \Pi(p)\, dG(p) + U\int_{\hat{p}}^{1} dG(p) \quad \text{using Figure 4 \& definition of } V(p)$$

$$= U + \int_{\underline{p}}^{\hat{p}} [\Pi(p) - U]\, dG(p) \qquad \text{since } \left(\int_{\underline{p}}^{\hat{p}} + \int_{\hat{p}}^{1}\right) dG(p) = 1$$

$$= U - \int_{\underline{p}}^{\hat{p}} G(p)\Pi'(p)\, d(p) \qquad \text{using } \Pi(\hat{p}) = U, G(\underline{p}) = 0, \text{and integration by parts}$$

$$= U + (1 - \beta\hat{p})(r - c)\int_{\underline{p}}^{\hat{p}} \frac{G(p)dp}{(1-\beta p)^2} \text{ subsituting from (A3)} \qquad \text{(A5)}$$

Putting (A4) and (A5) together and rearranging terms yields:

$$\beta^c U = u_o + \alpha\beta(1 - \beta\hat{p})(r - c)\int_{\underline{p}}^{\hat{p}} \frac{G(p)dp}{(1-\beta p)^2} > u_o. \qquad \text{(A6)}$$

Since $\beta^c U > u_o$ from Equation (A6), $\beta^c U = \pi_e(\hat{p})$ in Equation (A1), and $\pi_e'(p) < 0$, we know: $\pi_e(p) \geq \pi_e(\hat{p}) = \beta^c U > u_o$ or $\pi_e(p) > u_o$ for $\forall p \leq \hat{p}$; otherwise no firm is willing to invest and they will wait to obtain u_o. In particular, if $p = 1/2 \in [\underline{p}, \hat{p}]$, then we know from $\pi_e(1/2) > u_o$ that $2(R_o - u_o) > r + c$ which is useful in Equation (13). Combining Equation (A1) with Equation (A6) to eliminate U yields the optimality condition Equation (6) in the paper.

(III) Analyze the property of $\hat{p}_f$ in Equation (6)

By omiting μ, using $G(\underline{p}) = 0$, and integrating by parts, $\hat{p}_f$ in Equation (6) can be written as:

$$\hat{p}_f \equiv \alpha\beta(1 - \beta\hat{p})\int_{\underline{p}}^{\hat{p}} \frac{G(p)dp}{(1-\beta p)^2} = \alpha(1 - \beta\hat{p})\int_{\underline{p}}^{\hat{p}} G(p)\, d\left(\frac{1}{1-\beta p}\right)$$

$$= \alpha(1 - \beta\hat{p})\left[\frac{G(\hat{p})}{1-\beta\hat{p}} - G(\hat{p})\int_{\underline{p}}^{\hat{p}} \frac{1}{1-\beta p}\, d\left(\frac{G(p)}{G(\hat{p})}\right)\right]$$

$$= \alpha\hat{G}\left[1-(1-\beta\hat{p})E_{p\le\hat{p}}\left(\frac{1}{1-\beta p}\right)\right]$$

$$= \alpha\hat{G}\left[1-(1-\beta\hat{p})\mu_{T_d}\right] > 0, \qquad \text{(easy to see that } \hat{p}_f \le 1) \tag{A7}$$

which appears in the paper as Equation (8), in which $(1-\beta\hat{p})\mu_{T_d} \equiv \hat{q}_1$ is the "discounted" probability of project failure adjusted for the project's mean duration.

Equation (8) can be written in a different way as follows:

$$\hat{p}_f = \alpha\hat{G}\left\{1-(1-\beta\hat{p})E_{p\le\hat{p}}\left[1+\beta p+(\beta p)^2+(\beta p)^3+\cdots\right]\right\}$$

$$= \alpha\hat{G}\left\{1-E_{p\le\hat{p}}\sum_{t=1}^{\infty}(\beta p)^{t-1}(1-\beta\hat{p})\right\}$$

$$= \alpha\hat{G}\left\{\beta\hat{p}-E_{p\le\hat{p}}\sum_{t=2}^{\infty}(\beta p)^{t-1}(1-\beta\hat{p})\right\}$$

$$= \beta\alpha\hat{G}E_{p\le\hat{p}}\left(\frac{1}{1-\beta p}\right)(\hat{p}-p).$$

The last expression appears in the paper as Equation (9), in which project sustainability $\left(\frac{1}{1-\beta p}\right)$ and project preferability $(\hat{p}-p)$ move in the opposite directions for the same change in p since a more preferred project (riskier) is less likely to be sustained.

(IV) Prove Proposition 2

To perform comparative statics, we need to use the implicit function theorem, integration by parts, and the Leibniz rule:

$$\frac{d}{dt}\int_{b(t)}^{a(t)} f(x,t)dx = \int_b^a \frac{\partial f(x,t)}{\partial t}dx + \frac{da}{dt}f(a,t) - \frac{db}{dt}f(b,t).$$

Define $x_o \equiv R_o - u_o$ as the expected project return in excess of the reservation payoff. u_o is omitted from x_o in the text, but u_o is still kept in the Appendix for completeness. Differentiating

$\hat{p} = \hat{p}(c, r; \mu, \alpha, x_o)$ in Equation (6) with respect to the underlying parameters and applying these techniques yields the first-order derivatives of Equations (13), (14), and (15) in the paper.

Note that $R(p) \in [R_o, R(\underline{p})]$ for $p \in [\underline{p}, 1]$ since $R'(p) < 0$, where $R_o = pR(p)$ for all p so that $R_o = R(1)$ setting $p = 1$. Since the successful return $R(p)$ must exceed the interest cost r to have an incentive to borrow for investment, it must be that $R_o > r$. It then follows that $x_o > r - u_o$. Since $r > c$, we have $2R_o > (2x_o > 2r - 2u_o) > r + c - 2u_o$. Thus $2(R_o - u_o) > r + c$ or $\pi_e(1/2) > u_o$.

Differentiating $\frac{\partial \hat{p}}{\partial r}$ or $\frac{\partial \hat{p}}{\partial c}$ in Equation (13) with respect to c or r yields the third expression of Equation (13) in the paper.

If $r > R_o - u_o$, then $\frac{\partial^2 \hat{p}}{\partial r \partial c} < 0$ or $\frac{\partial}{\partial c}\left|\frac{\partial \hat{p}}{\partial r}\right| > 0$ under $2(R_o - u_o) > r + c$ and given $r > R_o - u_o > c$. Thus the (absolute value) slope of curve $\hat{p}$ with respect to r gets larger with higher c as in Figure 5.

If $r < R_o - u_o$, it is easier to have $2(R_o - u_o) > r + c$ but the sign of $\frac{\partial^2 \hat{p}}{\partial r \partial c}$ seems unclear from the third expression of Equation (13) under $R_o - u_o > r > c$. We can rely on the continuity of function $\hat{p}(c, r)$ and the monotone property of $\underline{r}_\mu(c)$ and $\bar{r}_\mu(c)$ to diagrammatically work out the sign of this cross effect as shown in Figure 5. Note that $r \in [\underline{r}_\mu(c), \bar{r}_\mu(c)]$ in Figure 5 is the range of feasible loan rates r given collateral c (tedious to derive, omitted here; ask us for the technical note via email if interested).

(V) Prove Proposition 3

The derivation of the first-order derivatives in Proposition 3 has been stated in (IV), and the following is the derivation of cross-derivatives presented in this Proposition.

Noticing $\frac{Y}{r-c} \equiv 1+\alpha\beta\int_{\underline{p}}^{\hat{p}} \frac{dG}{1-\beta p}$, differentiating this with respect to μ, and using the first expression in Equation (14), one sees:

$$\frac{-1}{\alpha\beta^2(r-c)}\frac{\partial Y}{\partial \mu} = \left[\frac{\alpha\hat{g}(r-c)}{Y}+1\right]\int_{\underline{p}}^{\hat{p}}\frac{\partial G}{\partial \mu}\frac{dp}{(1-\beta p)^2} < 0 \text{ since } \frac{\partial G}{\partial \mu} < 0.$$

Then, from $\frac{\partial Y}{\partial \mu} > 0$ and the first two expressions in Equation (13), we can derive the last two expressions of Equation (14) which means $\frac{\partial}{\partial \mu}\left|\frac{\partial \hat{p}}{\partial L}\right| < 0$ where $L \equiv r$ or c.

(VI) Prove Proposition 4

The method for the derivation of the first two results in this Proposition has been given in (IV).

To prove the third result in Proposition 4, use Equation (7) to have: $\frac{x_o}{r-c} - \hat{p}_f = \hat{p} + \frac{c}{r-c}$, and look at the total differential of $\hat{p}$ as follows:

$$\begin{aligned} d\hat{p} &= \frac{\partial \hat{p}}{\partial x_o} dx_o + \frac{\partial \hat{p}}{\partial \alpha} d\alpha \\ &= \frac{1}{Y}\left[x_o\frac{dx_o}{x_o} - (r-c)(1-\hat{q}_1)\hat{G}\alpha\frac{d\alpha}{\alpha}\right] && \text{using the first 2 derivatives in Eqn (15)} \\ &= \frac{r-c}{Y}\left[\frac{x_o}{r-c}\frac{dx_o}{x_o} - \hat{p}_f\frac{d\alpha}{\alpha}\right] && \text{using Eqn (8)} \end{aligned}$$

If $\frac{dx_o}{x_o} > \frac{d\alpha}{\alpha} > 0$, then $d\hat{p} > \frac{r-c}{Y}\left(\frac{x_o}{r-c}\frac{d\alpha}{\alpha} - \hat{p}_f\frac{d\alpha}{\alpha}\right) = \frac{r-c}{Y}\left(\frac{c}{r-c}+\hat{p}\right)\frac{d\alpha}{\alpha} > 0$ from Equation (7).

If $\frac{dx_o}{x_o} < \frac{d\alpha}{\alpha} < 0$, then $d\hat{p} < \frac{r-c}{Y}\left(\frac{x_o}{r-c}\frac{d\alpha}{\alpha} - \hat{p}_f\frac{d\alpha}{\alpha}\right) = \frac{r-c}{Y}\left(\frac{c}{r-c}+\hat{p}\right)\frac{d\alpha}{\alpha} < 0$, in which case, $\frac{|dx_o|}{x_o} > \frac{|d\alpha|}{\alpha}$.

(VII) Prove Proposition 5

Taking derivative of $\mu_T = 1/(\alpha\hat{G})$ and $\mu_{T'} = 1/\tilde{p}^c$ with respect to investment opportunities and firm types, we have:

$$\frac{\partial \mu_T}{\partial x_o} = -\frac{\hat{g}}{\alpha Y \hat{G}^2} < 0,\ \frac{\partial \mu_T}{\partial \mu} = -\frac{\partial \hat{p}}{\partial \mu}\frac{\hat{g}}{\alpha \hat{G}^2} < 0,\ \frac{\partial \mu_T}{\partial \alpha} = -\frac{1}{\alpha \hat{G}}\left(\frac{1}{\alpha} + \frac{\hat{g}}{\hat{G}}\frac{\partial \hat{p}}{\partial \alpha}\right) \lesseqgtr 0;$$

$$\frac{\partial \mu_{T'}}{\partial x_o} = \frac{1}{\left(\tilde{p}^c\right)^2 Y}\frac{d\tilde{p}}{d\hat{p}} > 0,\ \frac{\partial \mu_{T'}}{\partial \mu} = \frac{\partial \tilde{p}}{\partial \mu}\frac{1}{\left(\tilde{p}^c\right)^2} > 0,\ \frac{\partial \mu_{T'}}{\partial \alpha} = \frac{\partial \tilde{p}}{\partial \hat{p}}\frac{\partial \hat{p}}{\partial \alpha}\frac{1}{\left(\tilde{p}^c\right)^2} < 0. \qquad .$$

Here, $\frac{\partial \tilde{p}}{\partial \hat{p}} = \frac{\hat{g}}{\hat{G}}(\hat{p} - \tilde{p})$ >0. Under some minor condition, we have $\frac{\partial \tilde{p}}{\partial \mu} > 0$. The reason for $\frac{\partial \mu_T}{\partial \alpha}$'s ambiguous sign is that greater project availability reduces the waiting time (due to $1/\alpha > 0$) but firms' becoming choosier ($\frac{\partial \hat{p}}{\partial \alpha} < 0$) prolongs it.