

Loan Rates and the Two Faces of Collateral*

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Abstract

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Key Words Dynamic, Default risk, Collateral, Loan Rates.

JEL Classification C73, D83, G21

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Abstract

This paper analyzes the interaction between the loan rate and collateral in curbing default risk in loan contracts. It is shown that collateral may have a favorable or adverse incentive effect on default risk depending on the level of the loan rate. The loan rate and collateral are substitutable from the perspective of competing banks but they may be substitutes or complements from the perspective of borrowing firms. An equilibrium involving firms and banks exists only when the loan rate is low and when the loan rate and collateral are positively affected by firm default risk. The theory provides a general explanation for the fact that in practice there are few loans with high rates and that collateral and the loan rate are positively related when the loan rate is below a benchmark level. We test our theory using information from the U.S. Survey of Small Business Finances and show that the empirical results support our theory.

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1 Introduction

The use of collateral is pervasive in bank lending. Berger and Udell (1990) report that about 70% of all commercial and industrial loans in the U.S. are secured by collateral. Black, de Meza and Jeffreys (1996) find that 85% of loans to small businesses in the U.K. are subject to collateral provisions, and that start-ups of these businesses are particularly affected by their collateralizable wealth. Also, according to Black and de Meza (1992), the extensive use of collateral is responsible for the remarkably narrow spread of loan rates and for considerably low risk-premia (rarely exceeding 3-4%) on loans to small businesses despite their high rates of default. Moreover, empirical evidence shows that there is a positive relationship between collateral and the loan rate (See, Leeth and Scott 1989, 1992), and Berger and Udell (1990, 1995), among others).

Figure 1 depicts observations on the use of collateral and loan rates for firms in the U.S. Survey of Small Business Finances for the years of 1987, 1993 and 1998. Three basic patterns emerge from the figure. First, there is a positive association between posted collateral and loan rates when the loan rate is below a certain level (around 3%). Second, this positive relationship does not extend to loans with high rates. Third, there are few loans with high rates; only 5% of the sample loans yield rates exceeding 3%.¹

[Figure 1 is here]

These observations pose important and challenging questions about the use of collateral in relation to loan rates. Why is there a positive loan rate-collateral relationship when the rate is below a certain level? Why is not there a positive relationship when loan rates are high? and why are there so few loans at high interest rates? Current theories differ significantly in their predictions about the relationship between the loan rate and collateral;

¹The loan rate is measured as the premium over the bank's prime rate.

some postulate a positive collateral-loan rate relationship (see, for example, de Meza and Southey 1996, and Coco 1999) while others assert a negative one (see, for example, Boot and Thakor 1994, Besanko and Thakor 1987a, and Beaudry and Poitevin 1995). None of the existing theories is able to explain why there are so few loans at high interest rates and why the positive loan rate-collateral relationship occurs only at low rates. More importantly, it is not clear whether and how those observations are related

This paper provides an explanation for these observations. We develop a theory that differs sharply from existing ones in two important dimensions. First, our model considers an environment encompassing firm investment choice under uncertainty and informational asymmetries between banks and firms. Previous models have focused on the implications of cross-sectional informational asymmetries but ignored investment uncertainties faced by firms over time. A firm's dynamic investment choice involves the decision to invest in currently available projects or to wait for potentially better future opportunities. The tradeoff between investment opportunities over time together with the firm's informational advantage over the bank, affects the firm's loan repayment decision and the bank's lending policy.

Second, we investigate strategic interactions between banks and borrowers in a dynamic setting. The firm chooses a project with a certain level of risk for a given combination of collateral and the loan rate. Banks cannot observe the riskiness of any particular project to be financed but know the distribution of project risks. The firm's project selection strategy entails a decision rule whereby a project is undertaken (rejected) only when its type is below (above) some reservation level. The bank takes into account firms' decision rule as it sets the loan rate conditional on the level of collateral. Firms incorporate the bank's loan-rate policy in their selection of the amount of collateral. The decision rule of the firm in this setting gives the bank more information about loan risk than predicted by static models. The increase in the bank's information strengthens the incentive role of collateral and permits a reduction in loan rates. The strategic interaction between the bank and the firm in this dynamic setting

provides a rationale for the observed positive loan rate-collateral relationship at low interest rates.

Our model demonstrates that collateral has dichotomous incentive effects. Raising collateral exerts an adverse incentive effect on risk taking when the loan rate is high, but has a positive incentive effect when the rate is low. Since raising the loan rate creates a moral hazard problem, the loan rate has to be kept low to attenuate its own moral hazard effect and to induce a positive incentive effect from collateral. We prove that the moral hazard effect associated with collateral is less severe than with the loan rate when the rate is high. If the rate is set high already, the bank can trade more collateral (with a smaller adverse incentive effect) for a lower rate (with a larger positive incentive effect) to reduce the overall extent of moral hazard and mitigate loan losses. We find that the incentive effect of either contractual instrument, collateral or the loan rate, is stronger for riskier borrowers. Thus raising collateral or lowering the loan rate is more effective in restraining risk-taking by low-quality borrowers than high-quality borrowers.

We further show that the loan rate and collateral may be substitutes or complements from the firm's perspective, depending on whether loan rates are high or not. Substitutability (as in Bester, 1985) and complementarity (as in Stiglitz and Weiss, 1992) are analyzed separately in the literature without bridging the gap between these two relations. We find that there exists a cut-off loan rate above which there is substitutability and below which there is complementarity between the loan rate and collateral. The cut-off loan rate highlights the dichotomy of collateral incentives and also provides a different interpretation of the *single-crossing property*. Safer borrowers are more "interest-rate averse" while riskier ones are more "collateral averse" if the interest rate is above this cut-off level, whereas safer borrowers are more "collateral averse" while riskier ones are more "interest rate averse" if the rate is below the cut-off level.

Finally, we show that separation of borrower types is attainable through an incentive

mechanism other than the traditional signalling effect of collateral. An equilibrium at low loan rates is achieved through interactions between the bank and the firm under rational expectations. The optimal contract induces self-selection among firms since they have no incentive to take on more risk in this favorable situation of low loan rates. As a result, both the loan rate and collateral are positively related to firm default risk at the low loan rate equilibrium.

Our theory provides an explanation for the following facts: banks are reluctant to charge high rates, and collateral is positively correlated with the loan rate when rates are low.² We test the predictions of our theory using data from the Survey of Small Business Finances for the years of 1987, 1992 and 1998. We find that there exists a cut-off loan rate (around 3%), below which collateral and the loan rate are significantly positively correlated and above which there is no systematic relationship between these two loan contract instruments. Thus, the empirical results support the predictions of our theory.

The rest of the paper is structured as follows. Section 2 presents a model of the firm's investment choice. Section 3 analyzes the sensitivity of the firm's repayment decision to changes in credit contract terms. Section 4 examines the bank's lending policy. Section 5 investigates the firm's behavior of posting collateral and derives the relationship between the loan rate and collateral. Section 6 presents empirical evidence and Section 7 concludes the paper.

2 The Firm's Dynamic Investment Selection

This section develops a dynamic model to formulate the investment decision of a forward-looking firm under (*ex ante*) uncertainty about project outcomes. The firm borrows to

²One possible explanation for the paucity of high-rate loans is the constraints imposed by social norms and usury laws. While that explanation cannot be excluded, the avoidance of high rate loans is the product of interactions between banks' and firms' optimal choices in our model.

finance production. A *credit contract* is defined by collateral and interest payments. The acceptability of a project is affected by both its riskiness, (*ex post*) observable only to the firm, and the nature of the credit contract negotiated with the financing bank. If the project is not risky enough to be profitable under the contract, the firm waits for the arrival of a future project yielding higher profits. If the firm accepts the project and obtains a loan from the bank, then both parties get matched. After one period of production if the project succeeds, the firm and bank remain matched; if it fails, then they separate and must wait a further period before possibly reaching a new contract.

A firm's project yields a random return \mathbf{R} each period (sales revenue less non-capital costs), producing $R(p)$ with probability p and zero with probability $p^c \stackrel{\Delta}{=} 1 - p$. We follow the literature by assuming that different projects have the same mean return:

$$E(\mathbf{R}) = pR(p) \equiv R_o \quad \text{for } \forall p \in [\underline{p}, 1] \text{ and } \underline{p} > 0.$$

As shown in Appendix (A1), in this setting projects differ only in their riskiness, and riskier projects produce higher returns to the firm if they are successful.

The success probability of a project, \mathbf{p} , is assumed to be *i.i.d.* over time ³. The distribution of project types $\mathbf{p} \in [\underline{p}, 1]$ is $P(\mathbf{p} \leq p) = G(p)$, with density $g(p)$, mean μ and variance σ^2 . This distribution is firm-specific, and depends on firm risk type $\mu^c \stackrel{\Delta}{=} 1 - \mu$ which varies with the nature of business and the degree of prudence of the firm. We shall at times suppress μ from $G(p | \mu)$. An informational asymmetry arises since the firm observes the project type p (the *ex post* realization of \mathbf{p}) when seeking a loan to undertake the project, but the bank knows only its distribution $G(p)$. What type of project, \mathbf{p} , will arrive in the future is unknown *ex ante* even to the firm, and this uncertainty affects the firm's current

³The distribution G is assumed to be *i.i.d.* because all market participants believe that the market can be characterized by $G(p, t) = G(p)$ for all p and t , which turns out to be true in steady state. A similar rational expectations assumption is used in Burdett and Coles (1997) to discuss agents' stationary strategies.

investment decision.

For simplicity, we assume that all projects have the same total cost which is greater than the firm's (per-period) endowment ϖ . Project financing is restricted to a bank loan and is secured by collateral (no greater than ϖ); each firm carries out one project per period by borrowing one unit of credit. Banks provide financing for the projects that are accepted by firms, but old debt borrowed for a successful project has to be repaid before new debt can be serviced to continue the project. If a firm defaults on its loan in the event of project failure, it will lose its pledged collateral to its financing bank. We treat collateral as a contractual variable observable to both the firm and the bank. The firm has a continual need for borrowing since income from previous projects is not high enough to self-finance subsequent projects.

The random profit $\boldsymbol{\pi}$ to a firm each period is given by $R(p) - r$ with probability p and $-c$ with probability p^c , where r is the (gross) interest rate paid by the firm if the project succeeds and c is the value of collateral claimed by the bank when the firm is in default. The mean profit function is:

$$E(\boldsymbol{\pi}) = R_o - (pr + p^c c) \triangleq \pi_e(p).$$

As shown in Appendix (A1), under the assumption that the collateral is insufficient, $c < r$, riskier projects are more profitable to firms given a contract (c, r)

A firm's value function $V(p)$ is the present value of the decision on whether or not to accept a type p project this period given other investment opportunities in subsequent periods. $V(p) = \max\{\Pi(p), U\}$; $\Pi(p)$ and U are specified below.

$\Pi(p)$ denotes the value to the firm of accepting a type p project this period and behaving optimally later on. This value is determined by a recursive equation:

$$\Pi(p) = p\{[R(p) - r] + \beta\Pi(p)\} + p^c\{-c + \beta U\}, \quad (1)$$

where β is a time discount factor. If the project is a success producing profits $[R(p) - r]$ (with probability p) this period, it will be continued in the next period and the process generating $\Pi(p)$ will repeat itself. If the project fails and the collateral c is lost (with probability p^c) this period, the firm will have to re-enter the market (by posting another asset as collateral ($<\varpi$)) next period and receive a waiting value U from then on. Rearranging (1) yields:

$$\Pi(p) = \frac{\pi_e(p) + \beta p^c U}{1 - \beta p}. \quad (2)$$

As shown in Appendix (A1), firms tend to accept risky projects given a credit contract that is not fully collateralized.

U denotes the rejection (or waiting) value accruing to the firm that rejects the type p project this period and waits for a possibly better future opportunity. Suppose that the firm has a reservation payoff u_o in each waiting period; this can be viewed as a risk-free return on the endowment ϖ . Assume that a project is encountered with a probability α each period. If it comes across a type \mathbf{p} project next period, the firm chooses between taking a new value $V(\mathbf{p})$ from then on and continuing to wait. Thus, U is given by:

$$U = u_o + \beta [\alpha EV(\mathbf{p}) + \alpha^c U], \quad (3)$$

where $\alpha^c \triangleq 1 - \alpha$ and the expectation E is taken with respect to G , capturing project uncertainty.

Substituting (3) and (2) into the firm's valuation function yields Bellman's functional equation:

$$V(p) = \max \left\{ \frac{\pi_e(p) + \frac{\beta p^c}{1 - \alpha^c \beta} [u_o + \alpha \beta EV(\mathbf{p})]}{1 - \beta p}, \quad \frac{u_o + \alpha \beta EV(\mathbf{p})}{1 - \alpha^c \beta} \right\}, \quad (4)$$

which is used by the firm to determine its reservation-type policy \hat{p} : to accept any project of

type $p \leq \hat{p}$ without delay; to reject any project of type $p > \hat{p}$ and wait for better opportunities. This optimal project selection strategy is depicted in Figure 2⁴. The safest projects of types $p > \hat{p}$ are effectively rationed out of the borrower's investment consideration under the credit contract (c, r) (embedded in $\pi_e(p)$).

[Figure 2 is here]

3 The Firm's Loan Repayment Behavior

This section solves the firm's investment model, provides comparative static results with respect to changes in credit terms and firm quality, and determines the acceptability of loan rates to firms for any level of collateral. The results show that safer firms are more sensitive to loan rate changes given collateral and that a change in collateral has a dichotomous effect on project selection. The results also establish a close link between the firm's loan repayment and project investment decisions.

The loan rate and collateral in equation (4) affect the firm's reservation policy. We can identify the range of acceptable loan rates given a level of collateral, $\Omega_\mu(c) = [\underline{r}_\mu(c), \bar{r}_\mu(c)]$, so that reservation type \hat{p} falls within the project population $[\underline{p}, 1]$. A riskier firm is assumed to have a more dispersed population such that $\frac{d\hat{p}(\mu)}{d\mu^c} < 0$. Since the expected return on a project must be higher than the reservation payoff, we assume that $R_o > u_o$. Appendix (A2)

⁴If a credit contract is fully collateralized in the sense of $c > r$, then $\pi'_e(p) > 0$ and $\Pi'(p) > 0$ (seen from (16) in Appendix (A1)), so that the firm's optimal strategy will be to avoid default by accepting any project of type $p \geq \hat{p}$ and rejecting any project of type $p < \hat{p}$. Thus, sufficient collateralization reverses the firm's risk-taking behavior that initially arises from the fact that $R'(p^c) > 0$ (riskier projects, if successful, give higher returns). The possibility of credit rationing can then be ruled out altogether even with asymmetric information in the credit market.

shows that

$$\begin{aligned}\bar{r}_\mu(c) &= \frac{R_o - u_o - \underline{p}^c(\mu)c}{\underline{p}(\mu)} \propto (1/c, 1/\mu), \\ \underline{r}_\mu(c) &= \frac{R_o - u_o + \xi c}{1 + \xi} \propto (c, \mu),\end{aligned}\tag{5}$$

where ξ is defined in Appendix (A2). Thus, we find that the upper bound loan rate \bar{r} is inversely related to μ and c while the lower bound \underline{r} is positively related to μ and c ; the sensitivity of a firm to the loan rate is linked to its type and collateral.

Intuitively, each firm has a different upper bound of acceptable loan rates given collateral, and a firm will not borrow if the loan rate rises above that bound. Conversely, a firm is willing to undertake any type of project (including the safest one) if the loan rate reaches or falls below the lower bound, given collateral. Since this upper bound is greater for a lower type firm, risky firms are willing to pay higher loan rates than safe firms. Furthermore, safer firms are more sensitive to loan rate changes; they are more averse to high rates and more prudent at low rates. In contrast, riskier firms care less about high rates and are less responsive to low rates. The upper (lower) bound on the loan rate for any type of firm varies negatively (positively) with the level of collateral, implying that raising collateral lowers the acceptability of high rates and makes firms more prudent at low rates.

As derived in Appendix (A3), the optimality condition for the firm's investment selection is

$$\frac{R_o - u_o - c}{r - c} = \hat{p} + \alpha\beta(1 - \beta\hat{p}) \int_{\underline{p}}^{\hat{p}} \frac{G(p | \mu) dp}{(1 - \beta p)^2} > 0,\tag{6}$$

which determines type μ firm's reservation project type for the contract (c, r) , $\hat{p} = \hat{p}(c, r; \mu)$. As shown in Appendix (A4), (6) yields the following comparative static results about the

effects of changes in credit contract terms and in firm riskiness:

$$\begin{aligned}
\frac{\partial \hat{p}}{\partial c} &< \text{ (or } >) 0, \text{ if } r > \text{ (or } <) R_o - u_o; & \frac{\partial \hat{p}}{\partial r} &< 0; \\
\frac{\partial \hat{p}}{\partial \mu^c} &< 0; & \left| \frac{\partial \hat{p}}{\partial r} \right| &> \left| \frac{\partial \hat{p}}{\partial c} \right|; \\
\frac{\partial^2 \hat{p}}{\partial \mu \partial c} &> \text{ (or } <) 0, \text{ if } r > \text{ (or } <) R_o - u_o; & \frac{\partial^2 \hat{p}}{\partial \mu \partial r} &> 0.
\end{aligned} \tag{7}$$

This is illustrated in Figure 3 ⁵. We have assumed $\frac{\partial G(p|\mu)}{\partial \mu} < 0$, which implies that G first-order stochastically dominates (denoted \succ^{fbsd}) its lower mean counterparts. We thus arrive at the following theorem:

[Figure 3 is here]

Theorem 1 *A higher loan rate leads a firm to make a riskier investment decision by decreasing its reservation type. The use of collateral induces firms to become riskier in their project selection if the loan rate is high ($> R_o - u_o$), and to become more prudent if the loan rate is low ($< R_o - u_o$). Lower-quality firms take on riskier projects owing to their lower reservation types. The investment choice of any firm is more responsive to changes in the loan rate than to variations in collateral.*

While higher loan rates always create a moral hazard problem, the impact of greater collateral is dichotomous in that raising collateral may exert a positive incentive effect if the loan rate is low and an adverse effect otherwise. At low rates, firms try to take advantage of the low cost of borrowing by selecting safe projects and reducing the chance of default; moreover, there will be sufficient after-interest revenue left by successful projects. Since the

⁵It is easy to show $\frac{\partial \hat{p}}{\partial \alpha} < 0$, that is, a firm with more opportunities (higher α 's) becomes choosier about projects. In addition, $\frac{\partial^2 \hat{p}}{\partial \mu \partial c} > \text{ (or } <) 0$ for $r > \text{ (or } <) R_o - u_o$ and $\frac{\partial^2 \hat{p}}{\partial \mu \partial r} > 0$ for $\forall r$; this implies that the *marginal* sensitivity of investment choices to loan rates or collateral is greater for lower-quality firms. A rise in r causes all firms to take on more risk, and lower-quality firms will take even more risk; this is also the case for a rise in c if $r > R_o - u_o$. For $r < R_o - u_o$, a rise in c induces all firms to become prudent, and lower-quality firms will turn more prudent.

risk of failure is low in this case, firms do not increase their risk-taking but are willing to post collateral; more collateral adds to their cost of default and induces them to make more prudent investments.

At high loan rates, a firm has to choose a project which is risky enough so that the advantage of limited liability offsets the high cost of borrowing and earns the firm a profit. Since the chance of default is high in this case, the firm is reluctant to post collateral; raising the level of collateral would exacerbate the high cost of borrowing forcing the firm to take on more risk.

Firms differ in their business risks, and some are more prudent than others. These differences are reflected in their project choices, and different firms have different probabilities of repaying their loans. A firm's type is connected with its project types in a probabilistic manner. A safer (higher quality) firm is one with a safer distribution of projects. For $\mu_1 < \mu_2$, firm μ_1 is riskier than firm μ_2 in the sense that $G(p | \mu_2) \succ^{fbsd} G(p | \mu_1)$; such firm heterogeneity underlies the differences in the riskiness of firms' investment decisions $\hat{p}(c, r, \mu)$, as portrayed in Figure 3.

The average success probability, \tilde{p} , of projects accepted by a type μ firm is defined by

$$\begin{aligned} \tilde{p}(c, r | \mu) &= E(p | \underline{p} \leq p \leq \hat{p}) \quad \text{for } r \in \Omega_\mu(c) \\ &= \frac{1}{G(\hat{p}(c, r; \mu) | \mu)} \int_{\underline{p}(\mu)}^{\hat{p}(c, r; \mu)} pdG(p | \mu), \end{aligned} \tag{8}$$

where $\underline{p} < \tilde{p} < \min(\hat{p}, \mu)$. From the bank's perspective, $\tilde{p}(c, r | \mu)$ is firm μ 's average rate of loan repayment under a credit contract (c, r) . As shown in Appendix (A5), \tilde{p} is positively related to \hat{p} , and negatively to r and μ^c .

Comparing (7) and (22) (in (A5)), one sees that a firm's repayment behavior \tilde{p} is consistent with its investment decision \hat{p} , and that riskier firms have lower loan repayment rates.

Furthermore, the firm's repayment rate is more informative than its project selection rule of $\mathbf{p} \leq \hat{p}$ since its default risk \tilde{p}^c summarizes the randomness of its bankruptcy risks \mathbf{p}^c under this rule. Therefore, we will later use \tilde{p} rather than \hat{p} as the firm's reaction function relevant to the bank's lending decision.

The result in (7) (or (22)) shows that the incentive effect of collateral may be favorable or adverse, but less severe (if adverse) than that of the loan rate. This provides the bank with a desirable trade-off between the two instruments. Trading a collateral requirement for a lower loan rate reduces the overall extent of moral hazard, alleviates the degree of credit rationing required to curb default risk, and constitutes a significant improvement in repayment despite informational deficiencies faced by the bank. The repayment rate can be increased by raising collateral by the same amount as the decrease in loan rates when they are high, or by imposing collateral while keeping loan rates unchanged when they are low. Note that the dichotomy in collateral incentive effects arises only in a dynamic setting, as in our model.

4 The Bank's Loan-Rate Policy Conditional on Collateral

The preceding two sections emphasized the incentive role of collateral in affecting a firm's investment and repayment decisions. This section examines a risk-neutral representative bank's lending policy that is aimed at curbing informational problems and determining appropriate interest rates on its secured loans. Collateral reduces the risk exposure of informationally disadvantaged banks. There are many banks in the credit market and they are assumed to be identical Bertrand competitors. A bank maximizes its expected profit from lending by setting loan rates conditional on collateral. Competition results in zero expected

profits for banks in equilibrium with banks charging the same loan rate to firms that post equal levels of collateral. We find that the loan rate is inversely related to borrower quality if the quality can be learned, and that collateral can be traded for lower rates in equilibrium.

Suppose that funds are supplied by depositors to banks at a riskfree (gross) deposit rate $\rho_o > 1$, and that the amount supplied is unaffected by the loan rates the bank charges borrowers. Since project returns are the ultimate source of income to be allocated between firms and depositors, it is reasonable to assume that $R_o > \rho_o + u_o$. With full information on project type, the bank charges a Bertrand rate equal to ρ_o/p ; safer firms pay lower rates while riskier firms pay higher rates if their projects succeed.

When project type is private information of the firm, the bank has to charge a non-discriminating loan rate for all projects financed if there are no collateral requirements. Since the bank pools all loans to maintain its Bertrand zero-profit equilibrium, borrowers with very safe projects are much worse off as they are forced to heavily cross-subsidize borrowers with very risky projects in the loan pool. The moral hazard problem increases with rising loan rates due to the unobservability of project type to the bank, thus raising its incentive to ration credit. Adverse selection may occur because of the bank's inability to learn borrower type without collateral; this leads to market failure if the mix of borrowers is too risky for the Bertrand rate to exist. To boost the overall repayment rate in this case, the bank rations credit and lowers the loan rate.

The imposition of collateral on loans mitigates the extent of rationing due to moral hazard and can also eliminate the need for rationing caused by adverse selection. Firms may become less risky in their investments (if $r < R_o - u_o$ such that $\frac{\partial \tilde{p}}{\partial c} > 0$) since default implies losing their pledged collateral. As the bank is compensated for its loan loss by acquiring collateral in the face of default, the loan rate can be lowered to induce firms to act more prudently. If differences in collateral levels are correctly taken as signals of default risk or as means of self-selection, the bank can perfectly price-discriminate across borrowers. Differential

credit contracts remove cross-subsidization among different firms, reduce a firm's incentive to undertake risky projects, and induce safe borrowers to stay in the market. Although collateral may have a moral hazard effect of its own (if $r > R_o - u_o$ such that $\frac{\partial \tilde{p}}{\partial c} < 0$), the bank still relies on it to lower the loan rate since the overall degree of moral hazard is decreased (due to $\left| \frac{\partial \tilde{p}}{\partial c} \right| < \left| \frac{\partial \tilde{p}}{\partial r} \right|$). These assertions are proven in the analysis that follows.

Since the bank cannot observe project type, a profit or a loss may be made from financing a particular project even with collateral (i.e., $\check{R}(p; c, r | \mu) \triangleq rp + cp^c \gtrless \rho_o$ for a $p \leq \hat{p}$). However, in Bertrand equilibrium, the bank must earn zero expected profits from lending to a firm if collateral reliably reveals the firm's risk type or induces truth-telling. The bank's expected profit from lending to firm μ is given by:

$$E_{p \leq \hat{p}} \check{R}(p; c, r | \mu) = r\tilde{p} + c\tilde{p}^c - \rho_o \triangleq \check{R}(c, r | \mu),$$

where $\tilde{p}^c \triangleq 1 - \tilde{p}$. The Bertrand equilibrium implies:

$$\check{R}(c, r | \mu) = 0 \quad \text{or} \quad \tilde{p} = \frac{\rho_o - c}{r - c}. \quad (9)$$

Since $r > c$, one sees $\rho_o > c$. Also, it follows from $\tilde{p} < 1$ that $r > \rho_o$. Hence, $r > \rho_o > c$, i.e., the loan rate is greater than the deposit rate which is in turn higher than the amount of collateral required by the bank equilibrium. The permissible region for credit contracts is thus $(c < \rho_o) \times \Omega_\mu(c)$. Condition (9) determines the equilibrium loan rate for firm μ given c , denoted by $r^* = r^*(c, \mu, \rho_o)$ (or $r(c, \mu, \rho_o) |_{\check{R}=0}$, we suppress ρ_o later). This rate is a rational expectations equilibrium because the firm's repayment policy \tilde{p} has been incorporated as a reaction function into the bank's profit maximization decision.

The existence of an equilibrium rate can be established under certain sufficient conditions (as shown in Appendix (A6)), but we are interested only in the implication of the Bertrand

equilibrium $\check{R}^* = 0$ as a necessary condition. Since there is no restriction on the slope of the function $\check{R}(c, r | \mu)$ in this general setting, the competitive equilibrium defined by (9) need not be unique and some Bertrand rates may be Pareto dominated. The bank makes zero expected profits in any equilibrium and is indifferent between different equilibria, but firms investing in projects are better off if the loan rate is at a lower equilibrium level for a given amount of collateral. We will pick the smallest of these rates as the bank equilibrium r^* conditional on collateral; this implies $\check{R}_r^* > 0$ since the case of $\check{R}_r^* = 0$ can be ruled out ⁶. In Appendix (A6), we derive the properties of the bank's equilibrium rate outlined below:

$$\frac{\partial r^*(c, \mu)}{\partial \mu} < 0, \quad \frac{\partial r^*(c, \mu)}{\partial c} < 0, \quad \frac{\partial r^*(c, \mu)}{\partial \rho_o} > 0. \quad (10)$$

We arrive at the following theorem:

Theorem 2 *In Bertrand equilibrium among banks, safer firms are charged lower loan rates if firm type can be learned, increased collateral can be traded for lower loan rates, and a higher deposit rate leads to a higher loan rate.*

From the bank's perspective, the loan rate and collateral are substitutable in maintaining a zero-profit equilibrium. Comparing secured and unsecured loan contracts generates new insights into this issue. The Bertrand rate on an unsecured loan, r_o^* , is defined by the equation $r\tilde{p} = \rho_o$ while the Bertrand rate on a secured loan, r^* , is determined by the equation $r\tilde{p} + c\tilde{p}^c = \rho_o$ in (9). Substituting these rates into their defining equations, combining them and rearranging the result yields:

$$(r_o^* - r^*)\tilde{p}_o^* = r^*(\tilde{p}^* - \tilde{p}_o^*) + c\tilde{p}^* \quad (11)$$

where $\tilde{p}^* = \tilde{p}(c, r^* | \mu)$ and $\tilde{p}_o^* = \tilde{p}(r_o^* | \mu)$ are the average repayment rates of secured and

⁶ $\check{R}_r(c, r^* | \mu) = 0$ may hold for some c 's but cannot do so for $\forall c$.

unsecured loans, respectively.

Clearly, $r^* < r_o^*$ if $\tilde{p}^* > \tilde{p}_o^*$; that is, the reduction in loan rates is made possible by the imposition of collateral if the repayment of secured loans is higher than that of unsecured loans. However, $r^* < r_o^*$ is not sufficient to ensure $\tilde{p}^* > \tilde{p}_o^*$. Note that the substitution between the loan rate and collateral is required by the Bertrand equilibrium, but implies no increase in loan repayments. The ultimate improvement in repayments depends on the interaction between the bank's loan rate selection and the firm's collateral posting decision.

5 The Firm's Collateral Posting Decision

Having conducted a partial equilibrium analysis of firm investment and bank lending behavior, we close our model by incorporating a firm's collateral decision into a general equilibrium analysis. The bank's lending policy is taken as a reaction function in the firm's maximization problem under rational expectations. Integrating the firm's selection of projects and collateral with the bank's choice of interest rates on secured loans generates a simultaneous equilibrium as the optimal steady state outcome for interacting lenders and borrowers. We find that collateral as well as the loan rate are positively affected by firm default risk in the simultaneous equilibrium, which implies a positive correlation between the loan rate and collateral.

Taking as given the bank's lending rate policy $r^* = r^*(c, \mu)$, a risk-neutral firm of type μ chooses the optimal amount of collateral, c^* , and maximizes the intertemporal surplus from borrowing or waiting to invest:

$$\max_c \check{\Pi} = E \sum_{t=0}^{\infty} \beta^t y_t, \quad s.t. \quad \check{R}(c, r \mid \mu) = 0;$$

$$\text{where } y_t = \begin{cases} \pi_e(p_t), & \text{if } p_t \in [\underline{p}, \widehat{p}] \\ u_o, & \text{if } p_t \in (\widehat{p}, 1] \end{cases} \triangleq y(p_t), \quad (12)$$

where p_t is the type of project encountered in period t , and the firm's choice of project type has been incorporated into this problem. Since $p_t \sim G(p | \mu)$ is *i.i.d.*, we can drop the time subscript t to reduce (12) to:

$$\begin{aligned} \max_c \check{\Pi}(c, r; \mu) &= \frac{1}{\beta^c} \left[\int_{\underline{p}}^{\widehat{p}} \pi_e(p) dG(p) + \int_{\widehat{p}}^1 u_o dG(p) \right], \quad \text{s.t. } \check{R}(c, r | \mu) = 0; \\ \text{where, } \beta^c \check{\Pi}(c, r; \mu) &= (R_o - u_o - \rho_o) G[\widehat{p}(c, r; \mu) | \mu] + u_o = E_p[y(p)]. \end{aligned} \quad (13)$$

Let $c^* = c^*(\mu)$ denote the solution to this problem. Thus, $r^{**} = r^*(c^*(\mu), \mu)$ ($\triangleq r^{**}(\mu)$) denotes the bank's equilibrium lending rate corresponding to the firm's optimal choice of collateral. The pair (c^*, r^{**}) is then an optimal credit contract in the simultaneous equilibrium.

The gradient of the firm's surplus function, given below, is the direction in which $\check{\Pi}$ increases fastest:

$$\nabla \check{\Pi} = (\check{\Pi}_c, \check{\Pi}_r) = (R_o - u_o - \rho_o) * \widehat{g} * \left(\frac{\partial \widehat{p}}{\partial c}, \frac{\partial \widehat{p}}{\partial r} \right).$$

As shown by the arrows in Figure 4, the firm's surplus increases in the lower leftward direction if loan rates are high ($> R_o - u_o$), or in the lower rightward direction if loan rates are low ($< R_o - u_o$). At the loan rate $r = R_o - u_o$, the firm's surplus remains unchanged and the gradient points vertically down regardless of collateral levels. One of firm μ 's indifference curves, $\check{\Pi}(c, r; \mu) \equiv \bar{\pi}$, is denoted by $r = r^\pi(c, \mu)$ (or $r(c, \mu) |_{\check{\Pi} \equiv \bar{\pi}}$).

[Figure 4 is here]

The properties of the firm's indifference curve are derived in Appendix (A7) and are

outlined below:

$$\begin{aligned} \frac{\partial r^\pi(c, \mu)}{\partial c} &< (=, \text{ or } >) 0 && \text{if } r > (=, \text{ or } <) (R_o - u_o); \\ \frac{\partial r^\pi(c, \mu)}{\partial \mu} &< 0, && \frac{\partial^2 r^\pi(c, \mu)}{\partial c^2} = 0, && \frac{\partial^2 r^\pi(c, \mu)}{\partial \mu \partial c} > 0. \end{aligned} \quad (14)$$

The implications of (14) are stated below. First, all indifference curves are straight lines. If loan rates are high, indifference lines slope downward, suggesting substitutability between the loan rate and collateral. If loan rates are low, indifference lines slope upward, indicating complementary between the two instruments. The indifference line is horizontal at the loan rate $r = R_o - u_o$, showing no relation between the two instruments. This property, along with the gradient directions, implies that a lower-rate/higher-collateral contract can render a safe project more desirable when the loan rate is already low, whereas when the rate is high, both collateral and the loan rate have to be lowered to make a safe project more attractive. Second, for a given amount of collateral, a safer firm demands a lower loan rate, and this property is significant regardless of levels of the loan rate. Third, the single-crossing property is that if loan rates are high (low), the indifference line of a safe firm is flatter (steeper) than that of a risky firm whenever both lines cross only once. A safer firm is willing to post more collateral in exchange for a given reduction in loan rates when they are high, and will only accept a smaller increase in collateral for a given rise in loan rates when they are low. In other words, a riskier firm is more sensitive to collateral requirements if loan rates are high, and cares less about collateral if loan rates are low.

However, firm behavior is constrained by the loan-rate/collateral substitutability required by bank's equilibrium. A firm's collateral decision is affected by its interaction with the bank's secured loan-rate policy. As shown in Appendix (A8), an interior solution to firm μ 's collateral decision problem in (13) does not exist since none of the firm's indifference lines is tangent to the bank's zero-profit curve. Thus, (c^*, r^{**}) must be a corner solution in the

permissible region. As depicted in Figure 4 (proved in (A8)), the bank's zero-profit curve is steeper than any of the firm's indifference lines in the region with $r \geq R_o - u_o$; the firm's surplus increases by moving down the bank's zero-profit curve with a lower loan rate and greater collateral, and hence a simultaneous equilibrium involving firms and banks cannot occur in this region. In the region with $r < R_o - u_o$, a rational firm continues to move along the bank's zero-profit curve towards the lower boundary curve $\underline{r}_\mu(c)$ to maximize its surplus from posting collateral c_* and accepting a loan rate r^{**} ⁷. It is thus clear that posting collateral is surplus-improving for firms in equilibrium.

A comparative static analysis with respect to firm default risk (see Appendix (A8)) shows that:

$$\frac{dc^*}{d\mu^c} > 0, \quad \frac{dr^{**}}{d\mu^c} > 0, \quad \frac{dr^{**}}{dc^*} > 0. \quad (15)$$

This conclusion is based on the bank-borrower interaction and is presented in the following theorem:

Theorem 3 *In the simultaneous equilibrium that occurs as a corner solution in the low loan rate region, a firm with greater credit risk is subject to a higher loan rate and is willing to post more collateral to extract the maximum surplus from borrowing and investing. Therefore, the loan rate is positively correlated with collateral in equilibrium.*

[Figure 5 is here]

This theoretical result is consistent with conventional wisdom in banking practice and with the empirical evidence. Figure 5 also shows that a high-risk firm has no incentive of pretending to be a safe borrower for the purpose of lowering collateral, since its surplus would be reduced if it were to lie and take a credit contact designed for a safe firm. The surplus would be smaller at any interior point than on the lower bound, and a dishonest

⁷The bank's zero-profit curve may lie entirely within the region of $r < R_o - u_o$, and the corner solution still results.

firm will end up at an interior point; the loan rate would decrease too little to match the lowered collateral under the complementarity. In this sense, our model provides an incentive compatible contract that induces truth-telling or self-selection by firms ⁸. Additionally, repayment rates are higher with collateral than without it for $r < R_o - u_o$ (see (22)); hence the equilibrium interest rate must be lower for secured loans than for unsecured loans, as implied in (11).

The above theorem highlights the important incentive role of collateral in reducing default risk in equilibrium (which cannot occur at $r > R_o - u_o$). The joint use of collateral and the loan rate induces a favorable net incentive effect, since the *marginal* incentive effects of the two instruments are stronger for riskier firms and the moral hazard problem of collateral is less severe than that of loan rates when the rate is high. In this case, collateral (traded for lower rates) plays a significant role in limiting the level of moral hazard and in improving the risk mix of loans. However, firms can do better by deviating from this tradeoff. Given the smaller marginal rate of substitution between the loan rate and collateral for a firm than for a bank, the firm's greater willingness to exchange collateral increases for loan-rate reductions allows the bank to lower loan rates further until the rate is outside the region of $r > R_o - u_o$ and goes down to the region of $r < R_o - u_o$. As a result, the signalling role of collateral

⁸Our continuous-type model in (13) is equivalent in effect to the following framework, which consists of (i) the firm's incentive compatibility constraint:

$$\check{\Pi}(c(\mu_L), r(\mu_L); \mu_L) > \check{\Pi}(c(\mu_H), r(\mu_H); \mu_L)$$

(for any $\mu_L < \mu_H$), and (ii) the firm's reservation utility or rationality constraint:

$$\check{\Pi}(c(\mu), r(\mu); \mu) > u_o$$

(which holds since $\pi_e(p) > u_o$ for $p \leq \hat{p}$), and (iii) the bank's objective function:

$$\max_{c,r} \int_{\underline{\mu}}^{\bar{\mu}} \check{R}(c(\mu), r(\mu) | \mu) dF(\mu)$$

(where F is the distribution of the bank's customer types; this objective value equals zero under Bertrand competition). This new problem would give an equilibrium outcome: $c^* = c(\mu)$ and $r^* = r(\mu)$, which is similar to the simultaneous equilibrium generated by the original problem (13).

ceases to exist.⁹

6 Empirical Evidence

Our theory predicts that there is a cut-off loan rate ($R_o - u_o$) below which collateral and the loan rate are both positively correlated with loan risk. Moreover firms will not seek credit with a loan rate above that the cut-off level. If a loan is taken at a rate higher than that level, this reveals a non-optimal choice made by some borrower. Thus, there should be no systematic relationship between collateral, the loan rate and loan risk for high rate loans. As shown in Figure 1, only a small number of high rate loans are observed in practice and there is a positive relationship between collateral and the loan rate when the loan rate is low, suggesting that most firms make decisions consistent with the predictions of our model. This section provides a formal empirical test of our theory.¹⁰

To test the predictions of our theory, we draw information from the U.S. Survey of Small Businesses Finances (SSBF) for the years 1987, 1993 and 1998. The survey was conducted under the guidance of the Board of Governors of the Federal Reserve System and the Small

⁹To clarify this, consider the following. Suppose that the situation were reversed (e.g., the bank's zero-profit curve lies within the region of $r > R_o - u_o$ and is flatter than any firm's indifference lines). An inefficient sorting role for collateral would then emerge. In this setting, firms are reluctant to post as much collateral as required by the bank in exchange for a loan rate reduction, but are compelled to provide enough collateral as a signal of their type. Lower-risk firms may post more collateral to distinguish themselves from higher-risk ones even if this is value-reducing. Although this signalling role of collateral separates out borrowers by their default risks and the bank is able to offer differential credit contracts, moral hazard cannot be attenuated due to high loan rates; this will cause underinvestment and a negative loan rate-collateral correlation. This cannot happen in our framework.

¹⁰Berger and Udell (1990) investigate the risk-collateral relation using data from the "US Survey of Banking Lending Terms". The credit spread (i.e. the contractual loan rate minus a risk-free reference rate) is used in their study to capture the *ex ante* credit risk of individual borrowers. They find a positive correlation between the loan rate with collateral, suggesting a positive risk-collateral relationship. Berger and Udell (1995) analyze the determinants of the loan rate and the probability of a loan being collateralized on the basis of information from SSBF 1988. They take balance sheet ratios (e.g. leverage and profit margin) to be risk proxies, and find that loan risk relates positively to collateral and the loan rate. Leeth and Scott (1989) examine risk-collateral relations using information from the National Federation of Independent Businesses (NFIB). They treat the age of the firm as a (negative) proxy for its riskiness, and find that firm age is related negatively to collateral requirements.

Business Administration. It targeted non-financial, non-farm small business firms. The firms in the SSBF are relatively small (with less than 500 employees) and young (with the median age of 10 years). Since small and young firms usually do not have an established track record of repayments, potential lenders dealing with their loan applications face serious problems of asymmetric information. The SSBF is thus particularly suitable for testing our theory on the incentive role of collateral in alleviating problems due to informational asymmetries.

The SSBF data set includes interest rates and collateral requirements on firms' most recent loans. The survey indicates whether the loan rate is variable or fixed. Fixed rate loans could be substantially different in nature from variable rate loans. For example, Berger and Udell (1990, 1995) show that fixed rates are much stickier than variable rates. In our model, loan rates vary with firm default risks, so that it is more appropriate to use variable rates in an empirical test of our theory. We thus restrict our analysis to loans with variable rates in a manner similar to Berger and Udell (1995). The loan rate is measured as the premium over the bank's prime rate.

OLS and Logit models are used to examine the determinants of loan rates and the incidence of collateralization. The dependent variables are: *PREM*, representing the premium charged on the loan in the OLS regression, and *COLLAT*, which is equal to one if a loan is secured and zero otherwise in the Logit regression. The independent variables include three proxies for loan risk which are conventional and have been used in previous empirical work: (1) The logarithm of firm age, *LNAGE*, is used as a negative proxy for risk since the firms that have survived the previous years have a higher survival chance in subsequent years. (2) The logarithm of firm's total assets, *LNSIZE*, serves as a negative proxy for risk since smaller firms have a higher bankruptcy probability (Altman *et al* (1977)). (3) The logarithm of the length of loans, *LNLOANLENGTH*, measures the loan's time to maturity. The longer the time to maturity, the higher the probability of an adverse event causing the firm to default.

The other control variables include: (1) *LNLOAN SIZE*, loan size which is measured by the logarithm of the value of loans. Per-unit monitoring and administrative expenses of collateral decrease with rising loan sizes, which makes large loans secured more frequently than small loans (Jackson and Kronman (1979)); but the loan rate could also decrease with loan size due to economies of scale. (2) *LNRELATE*, the logarithm of the number of years during which the firm has had credit partnerships with its current lender. (3) The dummy variables, *CORPORATION*, *SUBSCORP*, *PARTNER* and *PROP* (omitted variable in the regressions), depend on whether a firm is non-subchapter corporation, subchapter corporation, partnership or sole proprietorship, respectively. (4) The dummy variable, *OWNER*, is equal to one if the firm is managed by its owner and to zero otherwise; (5) The dummy variable, *FAMILY*, is equal to one if 50% or more of a firm is owned by a single family and to zero otherwise. (6) Dummy variables, *CONSTRUCTION*, *SERVICES*, *RETAIL* and *OTHERIND*(omitted variable in the regressions), indicate whether the firm is in construction, in services, in retail or in other industries, respectively. (7) Year dummy variables.

Table 1 provides information on the means of the variables used in this study. Most of the variable means are quite stable over the period of the three surveys. For the sampled loans, 70% are collateralized, the average premium is 1.51%, and the average firm age is 22.2 years. The average size and time to maturity of loans are \$1,191,997 and 43.7 months, respectively.

[Tables 1 and 2 are here]

We first test for the determinants of loan rates and collateral requirements using the full sample without distinguishing between high rate and low rate loans. The results shown in Table 2 reveal that both the loan rate and the likelihood of a loan being collateralized are significantly related to loan risk. The coefficient for *LNAGE* is negative and significant at the 1% level in both the *PREM* and *COLLAT* regressions. The coefficient of *LNSIZE*

is negative and significant at the 5% level in the *PREM* regression, but not significant in the *COLLAT* regression. The coefficient of *LNLOANLENGHT* is positive and significant at the 1% level in the *COLLAT* regression, but not significant in the *PREM* regression. Since *LNAGE* and *LNSIZE* are negative proxies for risk and *LNLOANLENGHT* is its positive proxy, these results suggest that both the loan rate and the extent of collateralization are associated positively with loan risk; this is consistent with previous empirical findings.

Our theory asserts that the loan rate and collateral are positively related to default risk only when the loan rate is below a certain level. In order to test the prediction of our theory, we need to separate the sample of loans into low rate loans and high rate loans using the cut-off loan rate. In theory, the cutoff loan rate is equal to the excess of the average project return over the firm's reservation payoff, $R_o - u_o$. However, neither R_o nor u_o is directly observable. According to the theory, low rate loans are chosen by surplus maximizing firms. Such firms will not choose high rate loans so that high rate loans should be rarely observed. From Figure 1, we find that there is a large drop of the number of loans with premia above 2% and even fewer loans with premia higher than 3%. Based on this observation, we separate the sample loans into low rate loans with premia below or equal to 2% and high rate loans with premia over 2%. To test the robustness of the results, we also divide the loans into low rate loans and high rate loans by using the cutoff premium of 3%.

[Tables 3 and 4 are here]

Tables 3 and 4 separately test, for low rate loans and high rate loans, the determinants of loan rates and the incidence of a loan being secured. We find that loan risk significantly increases both the loan premium and the probability of collateralization for low rate loans. In contrast, there appears to be no relation between risk and the loan premium, or between risk and collateral, for high rate loans. Table 3 gives the results for a cutoff rate of 2%. In the *PREM* regression, the coefficients of *LNAGE* and *LNSIZE* are positive while the

coefficient of *LNLOANLENGTH* is negative. The results are all significant at the 1% level when the premium is lower than or equal to 2%. None of these coefficients is statistically significant when the premium exceeds 2%. In the *COLLAT* regression, the coefficient for *LNAGE* is positive while the coefficient for *LNLOANLENGTH* is negative, both being significant at the 1% level when the premium is below or equal to 2%; only the coefficient for *LNLOANLENGTH* is significant at the 5% level when the premium is above 2%. Similar results are found in Table 4 where sample loans are divided into high rate loans and low rate loans by using the cutoff premium of 3%. Thus, the results support the predictions of our theory. That is, positive relationships exist among the loan rate, collateral and risk when the loan rate is below a certain level; there is no link between loan risk and the loan rate, or between loan risk and collateral when the loan rate is higher.

7 Summary and Conclusion

This paper analyzed the interaction between the loan rate and collateral in curbing default risk in loan contracts. We demonstrated that increased collateral can always be traded for lower loan rates by banks at the partial equilibrium level. From firms' perspective, however, the relationship between the two instruments becomes more complicated, since the loan rate and collateral are substitutes if the rate is high and complements otherwise. Although a safer firm always demands a lower loan rate given collateral, different firms have different attitudes towards the use of collateral in different ranges of the loan rate. At high rates, a safer firm is less averse to collateral and is willing to post more collateral in exchange for a rate reduction. In contrast, at low loan rates, a riskier firm cares less about collateral and is willing to offer more collateral to match a rate increase. A firm's choice of projects and collateral interacts with the bank's selection of loan rates, and the use of collateral as a signalling device gives way to an incentive role of collateral in determining a simultaneous

equilibrium. Consequently, the optimal credit contract induces self-selection by firms, and both collateral and loan rates are positively affected by firm default risk at the low loan-rate equilibrium. Therefore, both credit instruments are positively correlated, and the fall in loan rates permitted by the use of collateral enhances loan repayments in equilibrium.

Our theory presented a plausible explanation for observations on the joint use of collateral and loan rates in lending contracts, using a framework that involves dynamic investment choices by firms, asymmetric information between banks and firms, and the endogenous choice of collateral. It provided a new theory to account for the paucity of loans at high interest rates and the mechanism governing the positive loan rate-collateral relationship when the rate is below a certain level.

Appendix

(A1) $Var(\mathbf{R})$ denotes the variance of \mathbf{R} . It follows from $E(\mathbf{R}) \equiv R_o$ that:

$$Var(\mathbf{R}) = R_o^2 \frac{p^c}{p}, \quad \frac{dVar(\mathbf{R})}{dp} = - \left(\frac{R_o}{p} \right)^2 < 0, \quad R'(p) = - \frac{R_o}{p^2} < 0. \blacksquare$$

If $r > c$, then $\pi'_e(p) = -(r - c) < 0$. Also, it follows from (17) that:

$$\Pi'(p) = - \frac{(r - c)(1 - \beta \hat{p})}{(1 - \beta p)^2} < 0, \quad \Pi''(p) = - \frac{2\beta(r - c)(1 - \beta \hat{p})}{(1 - \beta p)^3} < 0. \quad (16)$$

(A2) The loan-rate range for a given c , $\Omega_\mu(c) = [\underline{r}, \bar{r}]$, corresponds to $\hat{p} \in [\underline{p}, 1]$. Note that $\bar{r} = \bar{r}_\mu(c)$ is determined by setting $U = \Pi(\underline{p})$ and $V(p) = U$ for $\forall p \in [\underline{p}, 1]$. Substituting $V(p) = U$ to (3) leads to $\beta^c U = u_o$ (where $\beta^c \triangleq 1 - \beta$). Evaluating $\Pi(p)$ in (2) at \underline{p} , substituting it to $U = \Pi(\underline{p})$, yields $R_o - \underline{p}(\bar{r} - c) - c = \beta^c U$. Then, using $\beta^c U = u_o$, we obtain $\bar{r}_\mu(c)$ in (5), where $\bar{r} > R_o - u_o$ since $R_o - u_o \equiv \bar{c} > c$ as will be shown. \blacksquare

Note that $\underline{r} = \underline{r}_\mu(c)$ is determined by setting $U = \Pi(1)$ and $V(p) = \Pi(p)$ for $\forall p \in [\underline{p}, 1]$. Combining $U = \Pi(1)$ with (2) leads to $\beta^c U = R_o - \underline{r}$. Substituting $V(p) = \Pi(p)$ into (3) yields $(1 - \alpha^c \beta) U = u_o + \alpha \beta E \Pi(p)$. Noting $\hat{p} = 1$ and $E(p - \mu)^2 = \sigma^2$, recalling from (16) that $\Pi''(\mu) = -2\beta\beta^c(\underline{r} - c)(1 - \beta\mu)^{-3}$, second-order Taylor expanding $\Pi(p)$ in (2) around μ , and taking expectation with respect to p , we obtain:

$$\begin{aligned} (1 - \alpha^c \beta) U &\cong u_o + \alpha \beta E_p \left\{ \Pi(\mu) + \Pi'(\mu)(p - \mu) + \frac{1}{2} \Pi''(\mu)(p - \mu)^2 \right\} \\ &= u_o + \alpha \beta \left\{ \frac{R_o - \underline{r}\mu - c\mu^c + \beta\mu^c U}{1 - \beta\mu} - \frac{\beta\beta^c \sigma^2 (\underline{r} - c)}{(1 - \beta\mu)^3} \right\}. \end{aligned}$$

Let $\xi \equiv \frac{\alpha\beta\mu^c}{1 - \beta\mu}$. Substituting $\beta^c U = R_o - \underline{r}$ to the above yields:

$$\underline{r}_{(\mu, \sigma^2)}(c) = \frac{R_o - u_o + \left[\xi - \frac{\alpha\beta^2\sigma^2}{(1 - \beta\mu)^3} \right] c}{1 + \xi - \frac{\alpha\beta^2\beta^c\sigma^2}{(1 - \beta\mu)^3}},$$

which is reduced to the first-order Taylor expansion for $\underline{r}_\mu(c)$ in (5) if setting $\sigma^2 = 0$, though $\sigma^2 \neq 0$ actually. Note that σ^2 is small if p is concentrated around μ , and the Taylor approximation becomes better the smaller σ^2 gets. Clearly, $\underline{r} < R_o - u_o$ since $R_o - u_o > c$. Note that $\underline{r}_{(\mu, \sigma^2)}(c)$ may not necessarily be a linear function of c without the Taylor approximation since \hat{p} embedded in the expression of $\Pi(p)$ is not linear in c . From the above, $\underline{r} < R_o - u_o < \bar{r}$. ■

Noting that $\xi'(\mu) = -\frac{\alpha\beta\beta^c}{(1-\beta\mu)^2} < 0$ and recalling that $\underline{p}'(\mu) > 0$, one sees that:

$$\begin{aligned} \frac{\partial \bar{r}_\mu(c)}{\partial c} &= -\frac{\underline{p}^c}{\underline{p}} < 0, & \frac{\partial \underline{r}_\mu(c)}{\partial c} &= \frac{\xi}{1+\xi} > 0, \\ \frac{\partial \bar{r}_\mu(c)}{\partial \mu} &= -\frac{\underline{p}'(\mu)(R_o - u_o - c)}{\underline{p}^2(\mu)} < 0, & \frac{\partial \underline{r}_\mu(c)}{\partial \mu} &= -\frac{\xi'(\mu)(R_o - u_o - c)}{[1+\xi(\mu)]^2} > 0. \end{aligned}$$

By definition, $r \leq \bar{r}$ ensures $\hat{p} \geq \underline{p}$, $r = \bar{r}$ implies $\hat{p} = \underline{p}$, and $r > \bar{r}$ leads to $\hat{p} = 0$; also, $\hat{p} < 1$ when $r > \underline{r}$, and $\hat{p} = 1$ if $r \leq \underline{r}$. Additionally, $\Omega_{\mu_2}(c) \subset \Omega_{\mu_1}(c)$ for any given c if $\mu_1 < \mu_2$. Setting $\bar{r}_\mu(c) = \underline{r}_\mu(c)$ yields $\bar{c} = R_o - u_o$. Also, $\bar{r}_\mu(c) > \underline{r}_\mu(c)$ for $\forall c \in [0, \bar{c})$ because this leads to $0 < \underline{p}^c + \xi$, and going the other way round completes the proof.

(A3) From $\Pi(\hat{p}) = U$ (see Figure 2), it follows that:

$$\beta^c U = \pi_e(\hat{p}) \quad (= R_o - \hat{p}r - \hat{p}^c c \stackrel{\text{from (19)}}{>} u_o), \quad (17)$$

Hence, $r - c - \beta(R_o - c - \beta^c U) = (1 - \beta\hat{p})(r - c)$, and using this yields the sign of $\Pi'(p)$

in (16). Clearly, $\pi_e(p) > u_o$ for $p \leq \hat{p}$. Calculating U in (3) yields:

$$\begin{aligned}
& (\alpha\beta)^{-1} [(1 - \alpha^c\beta) U - u_o] \\
= & EV(p) \\
= & \int_{\underline{p}}^{\hat{p}} \Pi(p) dG(p) + U \int_{\hat{p}}^1 dG(p) \\
= & U + \int_{\underline{p}}^{\hat{p}} [\Pi(p) - U] dG(p) && \text{using } \Pi(\hat{p}) = U, G(\underline{p}) = 0 \\
& && \text{applying integration by parts} \\
= & U - \int_{\underline{p}}^{\hat{p}} G(p) \Pi'(p) dp && \text{substituting from (16)} \\
= & U + (1 - \beta\hat{p})(r - c) \int_{\underline{p}}^{\hat{p}} \frac{G(p) dp}{(1 - \beta p)^2}. && (18)
\end{aligned}$$

So, from (18) can we derive U :

$$\beta^c U = u_o + \alpha\beta(1 - \beta\hat{p})(r - c) \int_{\underline{p}}^{\hat{p}} \frac{G(p) dp}{(1 - \beta p)^2} > u_o. \quad (19)$$

Combining (17) with (19) to eliminate U yields the optimality condition in (6). Since $r > c$, (6) implies $R_o - u_o (= \bar{c}) > c$ for any firm μ . Let $c \in [0, \bar{c}] \triangleq \Lambda$. So, the domain $\Theta_\mu \triangleq \Lambda \times \Omega_\mu(c)$ for feasible (c, r) 's lies above the 45° line.

(A4) Let $\hat{G} = G(\hat{p})$. Applying the Leibniz rule, the implicit function theorem and the

technique of integration by parts to (6) yields:

$$\frac{\partial \hat{p}}{\partial c} = \frac{R_o - u_o - r}{(r - c)Y} \begin{cases} < 0, & \text{if } r > R_o - u_o \\ = 0 & \text{if } r = R_o - u_o \\ > 0, & \text{if } r < R_o - u_o \end{cases}, \quad \frac{\partial \hat{p}}{\partial r} = -\frac{R_o - u_o - c}{(r - c)Y} < 0, \quad (20)$$

$$\frac{\partial \hat{p}}{\partial \mu} = -\frac{r - c}{Y} \left\{ \alpha\beta (1 - \beta\hat{p}) \int_{\underline{p}}^{\hat{p}} \frac{\partial G}{\partial \mu} \frac{dp}{(1 - \beta p)^2} \right\} > 0,$$

where $\frac{\partial G(p|\mu)}{\partial \mu} < 0$ is assumed, $\mu_1 \stackrel{\leq}{\Rightarrow} \mu_2 \Rightarrow G(p | \mu_1) > G(p | \mu_2) \Leftrightarrow G(p | \mu_2) \stackrel{fosd}{\succ} G(p | \mu_1)$, and Y is defined as:

$$Y(c) = (r - c) \left(1 + \alpha\beta \int_{\underline{p}}^{\hat{p}} \frac{dG}{1 - \beta p} \right) > 0.$$

Since $r + c < 2(R_o - u_o)$ in $r \in \Omega_\mu(c)$ for $c < R_o - u_o$ and $r > c$, one sees that:

$$\frac{\partial \hat{p}}{\partial c} + \frac{\partial \hat{p}}{\partial r} = -\frac{1}{Y} < 0, \quad \frac{\partial \hat{p}}{\partial c} - \frac{\partial \hat{p}}{\partial r} = \frac{2(R_o - u_o) - (r + c)}{(r - c)Y} > 0,$$

$$\left| \frac{\partial \hat{p}}{\partial r} \right| > \left| \frac{\partial \hat{p}}{\partial c} \right| \text{ as } r > R_o - u_o, \quad \left| \frac{\partial \hat{p}}{\partial r} \right| > \frac{\partial \hat{p}}{\partial c} \geq 0 \text{ as } r \leq R_o - u_o.$$

(A5) Applying the L'hopital rule to (8) yields:

$$\begin{aligned} r \rightarrow \bar{r} &\Rightarrow \hat{p} \rightarrow \underline{p} \Rightarrow \tilde{p} \rightarrow \underline{p}, \\ r \rightarrow \underline{r} &\Rightarrow \hat{p} \rightarrow 1 \Rightarrow \tilde{p} \rightarrow \mu. \end{aligned} \quad (21)$$

Let $\hat{g} = g(\hat{p})$ and $\frac{\partial \tilde{p}(c, r | \mu)}{\partial \mu} = \frac{\partial \tilde{p}(\hat{p}(\cdot, \cdot; \mu) | \mu)}{\partial \mu}$. Differentiating (8) yields:

$$\hat{G} \frac{\partial \tilde{p}(c, r | \mu)}{\partial \mu} = \hat{g}(\hat{p} - \tilde{p}) \frac{\partial \hat{p}}{\partial \mu} + \int_{\underline{p}}^{\hat{p}} \frac{\partial g}{\partial \mu} p dp - \tilde{p} \frac{\partial \hat{G}}{\partial \mu}.$$

If \mathbf{p} is uniformly distributed on $[\underline{p}, 1]$, then $g(p) = \frac{1}{p^c}$ and $G(p) = \frac{p-\underline{p}}{p^c}$, with mean $\mu = \frac{1+\underline{p}}{2}$ and lower bound $\underline{p} = 2\mu - 1$. Rewrite the distribution with the mean as a parameter such that $g(p) = \frac{1}{2\mu^c}$ and $G(p) = \frac{p-2\mu+1}{2\mu^c}$. The parameteric effects on the distribution are $\frac{\partial g}{\partial \mu} = \frac{1}{2(\mu^c)^2} > 0$ and $\frac{\partial G}{\partial \mu} = -\frac{p^c}{2(\mu^c)^2} < 0$. Noticing that $\frac{\partial \hat{G}}{\partial \mu} < 0$ and $\hat{p} > \tilde{p}$ yields $\frac{\partial \tilde{p}(c,r|\mu)}{\partial \mu} > 0$. It is lengthy to prove the positive sign of $\frac{\partial \tilde{p}(\hat{p}(\cdot; \mu)|\mu)}{\partial \mu}$ in a general case by utilizing the *fosd* concept. We will not present the proof here due to space limitation. Contact us if interested. Since $\frac{d\tilde{p}}{d\hat{p}} = \frac{\hat{g}}{\tilde{G}} (\hat{p} - \tilde{p}) > 0$, one sees from (20) that:

$$\begin{aligned} \frac{\partial \tilde{p}}{\partial c} &= \frac{d\tilde{p}}{d\hat{p}} \frac{\partial \hat{p}}{\partial c} < (\text{or } >) 0, & \text{if } r > (\text{or } <) R_o - u_o; \\ \frac{\partial \tilde{p}}{\partial r} &= \frac{d\tilde{p}}{d\hat{p}} \frac{\partial \hat{p}}{\partial r} < 0, & \frac{\partial \tilde{p}}{\partial \mu^c} < 0, & \left| \frac{\partial \tilde{p}}{\partial r} \right| > \left| \frac{\partial \tilde{p}}{\partial c} \right|. \end{aligned} \quad (22)$$

(A6) Substituting from (5) and (21) yields the limiting values of \check{R} :

$$\begin{aligned} \lim_{r \rightarrow \bar{r}} \check{R}(c, r | \mu) &= \bar{r}\underline{p} + c\underline{p}^c - \rho_o = R_o - u_o - \rho_o > 0, \\ \lim_{r \rightarrow 0^+} \check{R}(c, r | \mu) &= c\mu^c - \rho_o < c - \rho_o < 0. \end{aligned} \quad (23)$$

From this and the continuity of $\check{R}(c, r | \mu)$ in $r \in \Omega_\mu(c)$ (the sufficient condition), one sees (by drawing a (r, \check{R}) -graph) that there exists at least one r such that $\check{R}(\cdot, r | \cdot) = 0$. Take the smallest of these r 's, if not unique, as r^* ; hence, we know $\check{R}_r^* \geq 0$ and rule out $\check{R}_r^* = 0$, where

$$\check{R}_r^* = \frac{\partial \check{R}(c, r | \mu)}{\partial r} \Big|_{r^*} = \tilde{p}^* + (r^* - c) \frac{\partial \tilde{p}(\hat{p}^* | \cdot)}{\partial \hat{p}} \frac{\partial \hat{p}(\cdot, r^*; \cdot)}{\partial r}.$$

■

The effects of c , μ and ρ_o on r^* are given by:

$$\begin{aligned}
\frac{\partial r^*}{\partial \mu} &= -\frac{\check{R}_\mu}{\check{R}_r} \Big|_{r^*} = -\frac{(r^* - c) \frac{\partial \tilde{p}(\tilde{p}^*(\cdot, \cdot; \mu) | \mu)}{\partial \mu} > 0}{\check{R}_r^* > 0} < 0, \\
\frac{\partial r^*}{\partial c} &= -\frac{\check{R}_c}{\check{R}_r} \Big|_{r^*} = -\frac{\tilde{p}^{*c} + (r^* - c) \frac{\partial \tilde{p}(\tilde{p}^*(c, \cdot; \cdot) | \cdot)}{\partial c}}{\check{R}_r^* > 0} \begin{cases} \geq 0, & \text{if } r > R_o - u_o \\ < 0, & \text{if } r \leq R_o - u_o \end{cases}, \\
\frac{\partial r^*}{\partial \rho_o} &= -\frac{\check{R}_{\rho_o}}{\check{R}_r} \Big|_{r^*} = \frac{1}{\check{R}_r} \Big|_{r^*} > 0, \quad \text{Note that } \rho_o \text{ is suppressed in } r^*.
\end{aligned} \tag{24}$$

If $\check{R}_r^* < 0$, then $\frac{\partial r^*}{\partial \mu} > 0$, which is unrealistic, implying that $\check{R}_r^* > 0$. One can see that $\check{R}_c^* > 0$ for $r > R_o - u_o$ under the uniform distribution, and hence $\frac{\partial r^*}{\partial c} < 0$. This is in general true as shown in (26) (in Appendix (A8)) with no need to rely on the uniform distribution assumption.

(A7) It follows from (20) that:

$$\begin{aligned}
r_c(c, \mu) \Big|_{\check{\Pi} \equiv \bar{\pi}} &= -\frac{\check{\Pi}_c}{\check{\Pi}_r} = -\frac{\partial \hat{p}}{\partial c} / \frac{\partial \hat{p}}{\partial r} = \frac{R_o - u_o - r [= r(c, \mu) |_{\check{\Pi} \equiv \bar{\pi}}]}{R_o - u_o - c} \\
&\begin{cases} < 0 & \text{if } r > R_o - u_o \\ = 0 & \text{if } r = R_o - u_o \\ > 0, (0 < r_c < 1 \text{ since } r > c) & \text{if } r < R_o - u_o \end{cases}, \\
r_\mu(c, \mu) \Big|_{\check{\Pi} \equiv \bar{\pi}} &= -\left(\hat{g} \frac{\partial \hat{p}}{\partial \mu} + \frac{\partial \hat{G}}{\partial \mu} \right) / \left(\hat{g} \frac{\partial \hat{p}}{\partial r} \right) < 0 \text{ (see below),} \\
r_{cc}(c, \mu) \Big|_{\check{\Pi} \equiv \bar{\pi}} &= 0, \quad r_{c\mu}(c, \mu) \Big|_{\check{\Pi} \equiv \bar{\pi}} = -\frac{r_\mu(c, \mu)}{R_o - u_o - c} > 0.
\end{aligned} \tag{25}$$

Since $y(p)$ in (12) is non-increasing (convex) and $G(p | \mu_2) \succ^{fbsd} G(p | \mu_1)$ due to $\mu_1 < \mu_2$, it follows (from the property of *fbsd*) that

$$E_{G(p|\mu_2)} [y(p)] < E_{G(p|\mu_1)} [y(p)], \quad \text{or} \quad \frac{\partial E_p [y(p)]}{\partial \mu} < 0.$$

This implies that $\frac{\partial}{\partial \mu} G[\widehat{p}(c, r; \mu) \mid \mu] = \widehat{g} \frac{\partial \widehat{p}}{\partial \mu} + \frac{\partial \widehat{G}}{\partial \mu} < 0$ (this can be verified under the uniform distribution). Hence, $r_\mu(c, \mu) \big|_{\check{\Pi} \equiv \check{\pi}} < 0$.

(A8) Substituting from (24) and (25) to $r_c(c, \mu) \big|_{\check{\Pi} \equiv \check{\pi}} = r_c(c, \mu) \big|_{\check{R} \equiv 0}$ leads to $R_o - u_o = \rho_o$ for all permissible (c, r) 's, a contradiction. ■

With $R_o - u_o > c$ and $\check{R}_r^* > 0$, substituting from (24) and (25) to $r_c(c, \mu) \big|_{\check{\Pi} \equiv \check{\pi}} > r_c(c, \mu) \big|_{\check{R} \equiv 0}$ yields $R_o - u_o > \rho_o$; going the other way round leads to the relation between the two slopes. Then,

$$\begin{cases} 0 > r_c(c, \mu) \big|_{\check{\Pi} \equiv \check{\pi}} > r_c(c, \mu) \big|_{\check{R} \equiv 0} & \text{if } r > R_o - u_o \\ r_c(c, \mu) \big|_{\check{\Pi} \equiv \check{\pi}} = 0 > r_c(c, \mu) \big|_{\check{R} \equiv 0} & \text{if } r = R_o - u_o \\ r_c(c, \mu) \big|_{\check{\Pi} \equiv \check{\pi}} > 0 > r_c(c, \mu) \big|_{\check{R} \equiv 0} & \text{if } r < R_o - u_o \end{cases}, \quad (26)$$

implying that the sign of $r_c(c, \mu) \big|_{\check{R} \equiv 0}$ for $r > R_o - u_o$ is clarified. Thus, $|r_c(c, \mu) \big|_{\check{\Pi} \equiv \check{\pi}}| < |r_c(c, \mu) \big|_{\check{R} \equiv 0}|$ for $r > R_o - u_o$. ■

To derive the point at which the curve $r(c, \mu) \big|_{\check{R} \equiv 0}$ intersects the curve $r_\mu(c)$ (in (5)), we use (21) to reduce the former curve to $\mu(r - c) = \rho_o - c$, ending up with a set of two equations:

$$\begin{cases} (1 + \xi)r - \xi c = R_o - u_o \\ \mu r + \mu^c c = \rho_o \end{cases}.$$

Solving this set yields $c^* = c^*(\mu)$ and $r^{**} = r^{**}(\mu)$. Substituting this solution back to the set, differentiating with respect to μ , and solving a Cramer system of two resulting equations, yields:

$$\frac{dc^*}{d\mu} = \frac{\Delta_c}{\Delta} < 0, \quad \frac{dr^{**}}{d\mu} = \frac{\Delta_r}{\Delta} < 0, \quad \frac{dr^{**}}{dc^*} = \frac{dc^*/d\mu}{dr^{**}/d\mu} = \frac{\Delta_c}{\Delta_r} > 0;$$

where $\xi > 0$, $\xi'(\mu) < 0$ (in Appendix (A2)), and

$$\begin{aligned}\Delta &= \xi\mu + (1 + \xi)\mu^c > 0, \\ \Delta_c &= (r - c)[\mu\xi'(\mu) - (1 + \xi)] < 0, \\ \Delta_r &= -(r - c)[\xi + \mu^c\xi'(\mu)] = -\frac{\alpha(\beta\mu^c)^2(r - c)}{(1 - \beta\mu)^2} < 0.\end{aligned}$$

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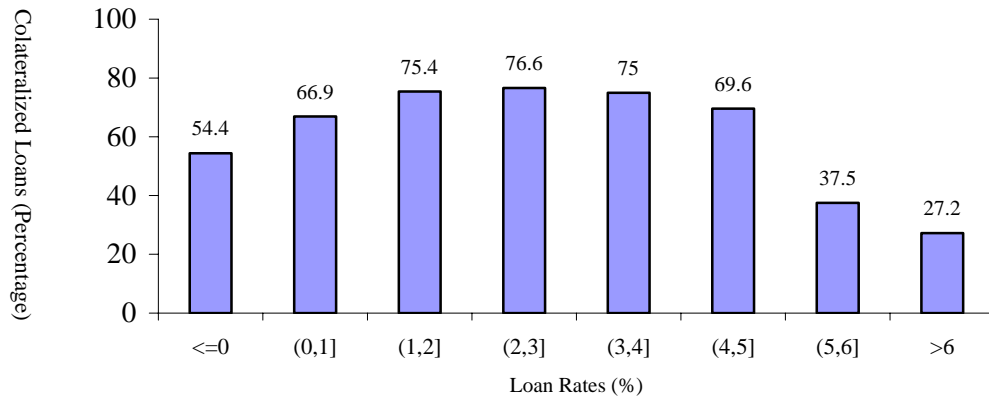
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Figure 1

The Use of Collateral and the Loan Rate in U.S. Small Businesses

The figures present the basic relationship between collateral and the loan rate in U.S. small businesses. The data is from the Survey of Small Business Finances for the year of 1987, 1992 and 1998. The top diagram shows the percentage of collateralized loans across different loan rates. The bottom diagram shows the number of loans in the sample across different loan rates. The loan rate is defined as the premium over the prime rate.

The Relationship Between the Loan Rate and the Use of Collateral



The Number of Loans with Different Loan Rates

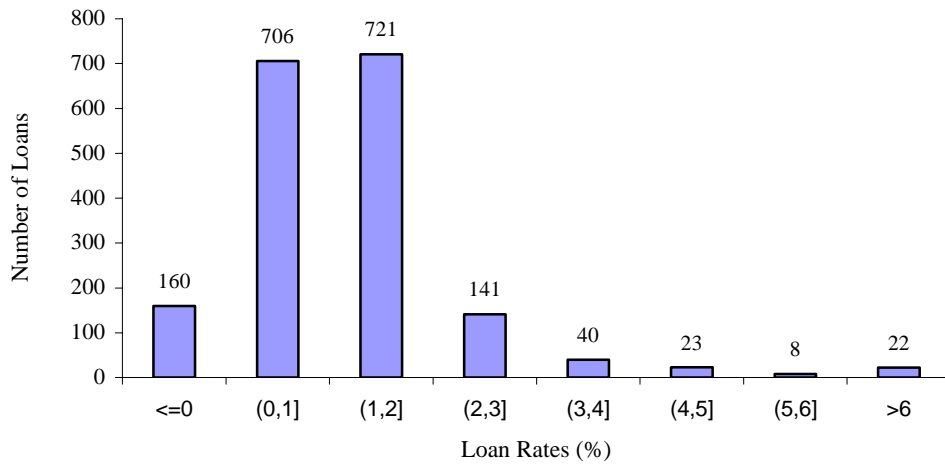


Figure 2
A Firm's Project Selection Decision

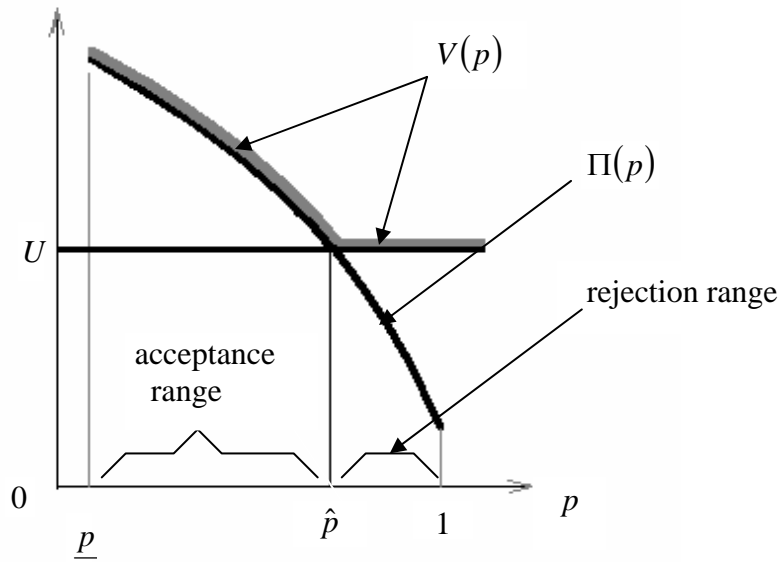


Figure 3
The Effect of Changes in the Loan Rate on Project Selection

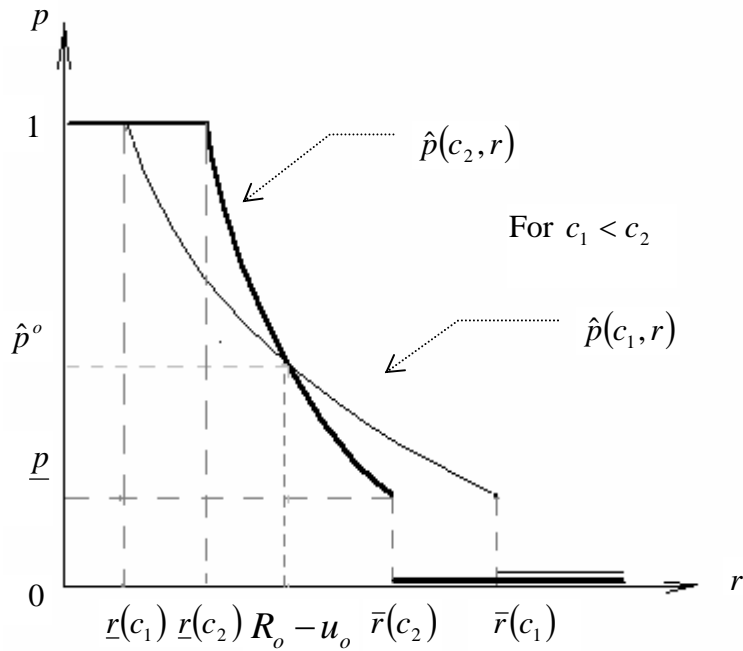


Figure 4
 The Existence of an Optimal Credit Contract in Equilibrium

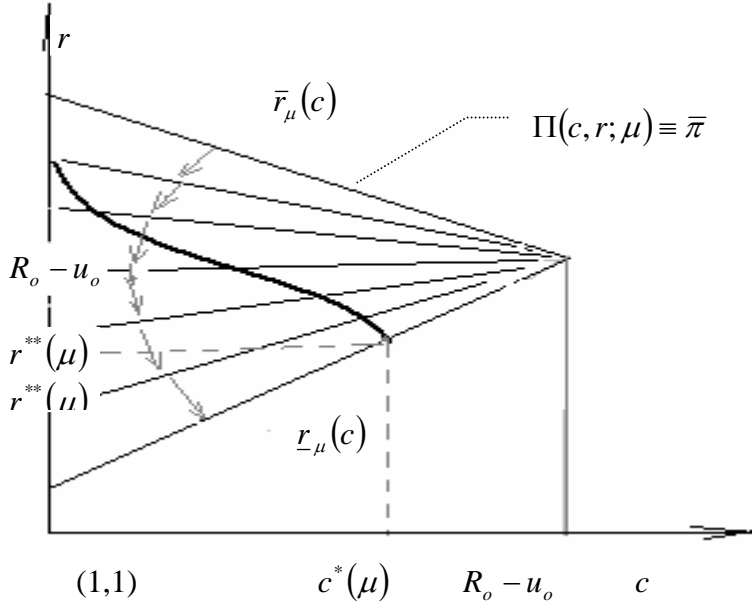


Figure 5
 The Positive Correlation between the Loan Rate and Collateral in Equilibrium

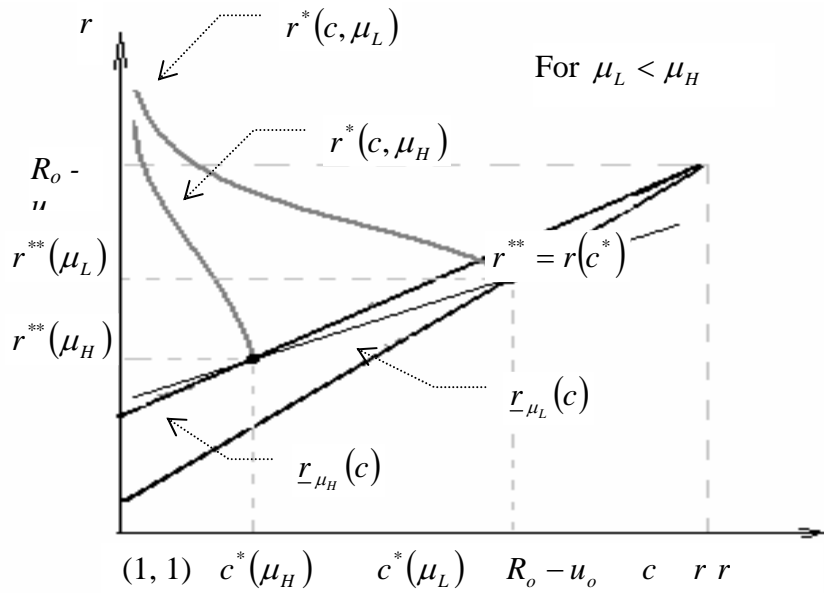


Table 1

Summary Statistics of the Variables Used in Regressions

This table provides summary statistics of the variables used in this study. The data come from the Survey of Small Business Finances 1987, 1993 and 1998 data bases. *PREM* is the premium charged on the loan. *COLLAT* is a dummy variable which is equal to 1 or 0 indicating if a loan is secured by collateral or not. The *LNAGE* is the logarithm of firm age. *LNSIZE* is the logarithm of firm size. *LNLOANLENGTH* is the logarithm of the length of loans. *LNLOANSIZE*, is the logarithm of value of loans. *LNRELATE*, is the logarithm of the number of years during which a firm has had credit partnerships with its current lender. *CORPORATION*, *SUBSCORP* and *PARTNER* are dummy variables indicating whether a firm is non-subchapter corporation, subchapter corporation, partnership or sole proprietorship, respectively. *OWNER* is a dummy variable which is equal to one if a firm is managed by its owner and to zero otherwise; *FAMILY* is a dummy variable which is equal to one if 50% or more of a firm is owned by a single family and to zero otherwise. *CONSTRUCTION*, *SERVICES*, *RETAIL* and *OTHERIND* are dummy variables indicating whether a firm is in construction, services, retail or other industries, respectively. N is the number of observations.

	1988		1993		1998		All Years	
	<u>N</u>	<u>Mean</u>	<u>N</u>	<u>Mean</u>	<u>N</u>	<u>Mean</u>	<u>N</u>	<u>Mean</u>
<i>PREM</i>	690	1.52	903	1.32	232	2.19	1825	1.51
<i>COLLATE</i>	686	0.70	903	0.70	232	0.65	1821	0.70
<i>LNAGE</i>	688	2.84	903	2.57	232	2.41	1823	2.65
<i>LNSIZE</i>	677	13.30	903	13.82	232	11.82	1812	12.15
<i>LNLOANLENGTH</i>	552	3.23	903	3.01	205	3.37	1660	3.12
<i>LNLOANSIZE</i>	690	11.84	903	12.46	232	11.82	1812	12.15
<i>LNRELATE</i>	670	2.04	878	1.75	184	1.42	1732	1.83
<i>CORPORATION</i>	690	0.56	903	0.53	232	0.36	1812	0.52
<i>SUBSCORP</i>	690	0.19	903	0.29	232	0.30	1812	0.25
<i>PARTNER</i>	690	0.09	903	0.07	232	0.10	1812	0.08
<i>OWNER</i>	690	0.87	903	0.75	232	0.88	1812	0.81
<i>FAMILY</i>	690	0.76	903	0.73	232	0.78	1812	0.75
<i>CONSTRUCTION</i>	690	0.04	903	0.06	232	0.06	1812	0.05
<i>SERVICE</i>	690	0.21	903	0.24	232	0.30	1812	0.24
<i>RETAIL</i>	690	0.29	903	0.20	232	0.19	1812	0.23

Table 2

The Determinants of the Use of Collateral and the Loan Rates

This table tests the determinants of the use of collateral and the Loan Rate. The data come from the National Survey of Small Business Finance 1987, 1993 and 1998 data bases. Column 2 reports the results on the choice of collateral using the Logit regression. Column 3 reports the results on the determinants of the loan rate using the OLS regression. The definitions of dependent and independent variables are the same as in Table 1. Standard Errors of the coefficients are reported in parentheses. ** and * indicate statistical significance at the 1 and 5 percent level, respectively.

	<i>COLLAT</i> (Logit)	<i>PREM</i> (OLS)
<i>LNAGE</i>	-0.2696** (0.084)	-0.1201** (0.042)
<i>LNSIZE</i>	0.0503 (0.053)	-0.0610* (0.027)
<i>LNLOANLENGTH</i>	0.6443* (0.064)	0.0203 (0.029)
<i>LNLOANSIZE</i>	0.2315** (0.052)	-0.0634* (0.026)
<i>LNRELATE</i>	-0.0971 (0.067)	-0.0966** (0.034)
<i>CORPORATION</i>	0.2087 (0.192)	-0.1903 (0.101)
<i>SUBSCROP</i>	-0.1770 (0.208)	-0.3122** (0.108)
<i>PARTNER</i>	-0.0865 (0.281)	-0.0933 (0.144)
<i>OWNER</i>	-0.0500 (0.158)	-0.0015 (0.080)
<i>FAMILY</i>	0.1701 (0.158)	-0.0176 (0.076)
<i>CONSTRUCTION</i>	-0.2768 (0.259)	-0.0486 (0.136)
<i>SERVICE</i>	-0.1027 (0.155)	-0.0840 (0.081)
<i>RETAIL</i>	0.1638 (0.158)	-0.0564 (0.079)
<i>CONSTANT</i>	-3.6301 (0.605)	3.8098 (0.301)
Control for Year Effect	Yes	Yes
Number Of Obs.	1565	1569
Pseudo R ²	0.111	0.104

Table 3

The Determinants of the Use of Collateral and the Loan Rate for Low and High Rate Loans

This table tests the determinants of the use of collateral and the loan rate for low rate loans and high rate loans. Low rate loans refer to loans with premia lower or equal to 2%. High rate loans refer to loans with premia higher than 2%. Columns 2 and 3 report the results on the choice of collateral for low and high rate loans using the Logit regression. Columns 4 and 5 report the results on the determinants of loan rates for low and high rate loans using the OLS regression. The definitions of dependent and independent variables are the same as in Table 1. Standard Errors of the coefficients are reported in parentheses. ** and * indicate statistical significance at the 1 and 5 percent level, respectively.

	COLLAT (Logit)		PREM(OLS)	
	PREM<=2%	PREM>2%	PREM<=2%	PREM>2%
<i>LNAGE</i>	-0.3627** (0.093)	0.2403 (0.240)	-0.0789** (0.024)	-0.2238 (0.161)
<i>LNSIZE</i>	0.0478 (0.057)	0.1153 (0.182)	-0.0517** (0.015)	-0.1289 (0.121)
<i>LNLOANLENGTH</i>	0.6824** (0.071)	0.5013* (0.177)	0.0453** (0.016)	0.0185 (0.109)
<i>LNLOANSIZE</i>	0.2467** (0.055)	0.1672 (0.169)	-0.0528** (0.015)	0.0742 (0.114)
<i>LNRELATE</i>	-0.0653 (0.073)	-0.1910 (0.190)	-0.0663** (0.019)	0.1177 (0.124)
<i>CORPORATION</i>	0.3960 (0.214)	-0.9427 (0.509)	0.0456 (0.058)	-0.3360 (0.352)
<i>SUBSCROP</i>	-0.1178 (0.228)	-0.4422 (0.593)	-0.1066 (0.062)	-0.1580 (0.390)
<i>PARTNER</i>	0.0764 (0.312)	-0.5110 (0.728)	-0.1007 (0.083)	0.1301 (0.487)
<i>OWNER</i>	-0.0019 (0.167)	-0.5472 (0.594)	-0.0057 (0.044)	-0.0957 (0.348)
<i>FAMILY</i>	0.2702 (0.167)	-0.4377 (0.490)	0.0245 (0.042)	-0.1128 (0.307)
<i>CONSTRUCTION</i>	-0.2702 (0.161)	-0.5884 (0.819)	0.0077 (0.075)	0.0090 (0.612)
<i>SERVICE</i>	0.0187 (0.168)	-0.6404 (0.469)	0.0080 (0.045)	-0.5842 (0.319)
<i>RETAIL</i>	0.1756 (0.170)	-0.1227 (0.492)	0.0739 (0.044)	-0.2017 (0.327)
<i>CONSTANT</i>	-3.958** (0.660)	-3.632 (1.860)	2.7458** (0.170)	5.1426** (1.186)
Control for Year Effect	Yes	Yes	Yes	Yes
Number of Observations	1364	201	1368	201
Pseudo/Adjusted R ²	0.120	0.191	0.132	0.197

Table 4

The Determinants of the Use of Collateral and the Loan Rate for Low and High Rate Loans: Robustness Tests

This table tests the determinants of the use of collateral and the loan rate the low rate loans and high rate loans. Low rate loans refer to loans with premia lower or equal to 3%. High rate loans refer to loans with premia higher than 3%. Columns 2 and 3 report the results on the choice of collateral for low and high rate loans using the Logit regression. Columns 4 and 5 report the results on the determinants of loan rates for low and high rate loans using the OLS regression. The definition of dependent and independent variables are the same as in Table 1. Standard errors of the coefficients are reported in parentheses. ** and * indicate statistical significance at the 1 and 5 percent level, respectively.

	COLLAT (Logit)		PREM (OLS)	
	PREM<=3%	PREM>3%	PREM<=3%	Prem>3%
<i>LNAGE</i>	-0.1809** (0.051)	0.1304 (0.228)	-0.0784** (0.026)	-0.571 (0.295)
<i>LNSIZE</i>	0.0256 (0.032)	0.2183 (0.176)	-0.0590** (0.017)	-0.3727 (0.224)
<i>LNLOANLENGTH</i>	0.3773** (0.037)	0.2841 (0.176)	0.0365* (0.018)	-0.1657 (0.216)
<i>LNLOANSIZE</i>	0.1384** (0.032)	0.1327 (0.167)	-0.0548** (0.016)	0.2298 (0.216)
<i>LNRELATE</i>	-0.0547 (0.042)	-0.1290 (0.181)	-0.0887** (0.021)	0.3757 (0.216)
<i>CORPORATION</i>	0.1400 (0.120)	-0.5571 (0.506)	0.005 (0.064)	0.6145 (0.634)
<i>SUBSCROP</i>	-0.1005 (0.129)	-0.4225 (0.574)	-0.1596* (0.068)	0.4277 (0.682)
<i>PARTNER</i>	0.0022 (0.176)	-0.5889 (0.756)	-0.0691 (0.091)	1.653 (0.919)
<i>OWNER</i>	-0.0288 (0.096)	-0.1017 (0.540)	-0.0024 (0.050)	0.0010 (0.639)
<i>FAMILY</i>	0.0968 (0.091)	0.4065 (0.457)	0.0202 (0.047)	-0.0514 (0.579)
<i>CONSTRUCTION</i>	-0.117 (0.160)	-1.085 (0.764)	-0.0505 (0.051)	0.3248 (1.052)
<i>SERVICE</i>	-0.0101 (0.095)	-0.9796* (0.454)	-0.0259 (0.051)	-0.9072 (0.553)
<i>RETAIL</i>	0.1154 (0.096)	-0.2374 (0.470)	0.0235 (0.050)	-0.8102 (0.581)
<i>CONSTANT</i>	-2.048** (0.366)	-4.952* (1.982)	3.1345** (0.189)	8.7602** (2.09)
Control for Year Effect	Yes	Yes	Yes	Yes
Number of Observations	1487	78	1491	78
Pseudo/Adjusted R ²	0.111	0.330	0.120	0.292